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Power Allocation for Shared Transponders

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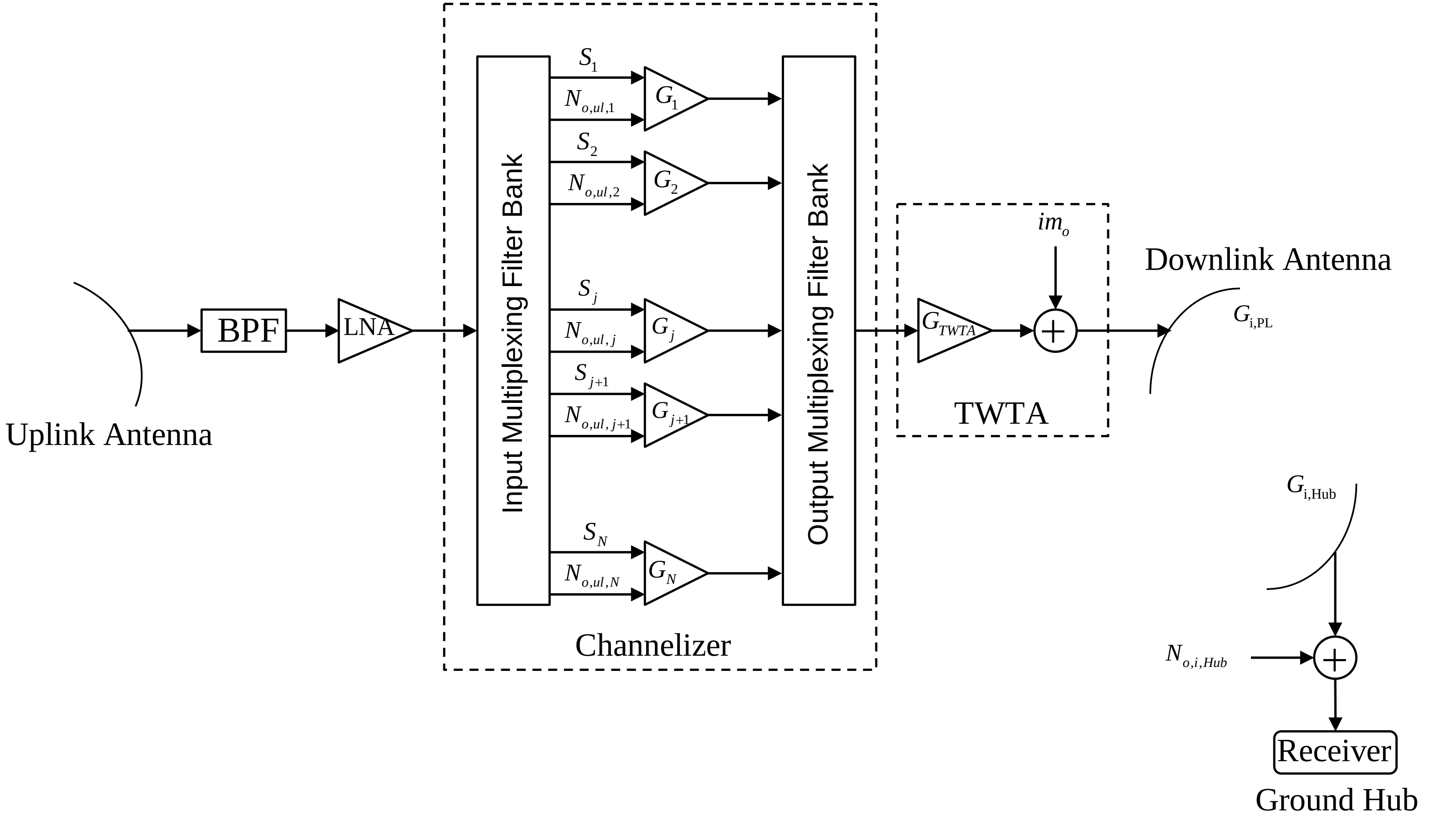
Integrity ★ Service ★ Excellence

Expected Contributions

- Design criteria addressing both technical and operational needs
  - Take a control-theoretic approach to capture meaningful technical and operational “corners” of the design space
- Technical constraints to stress
  - Satellite transponder nonlinearities
  - Multi-link terminals
  - Uplink noise jamming
- Operational focus
  - Draw benefits of power sharing among multi-carriers
  - Address current and expected operational gaps
  - Manage tradeoffs between link supportability

Selective Sub-Channel Gain Model



Ground Receiver LNA Input

- Downlink Gains

$$G_{D,i} = \frac{G_{i,PL} G_{i,Hub}}{L_{i,d,path} L_{i,d,rain} L_{i,d,misc}}, \qquad i = 1, \ldots, N$$

- Transponder Nonlinearities

$$im_o = \frac{IM_{in} P_{\max}}{BW_T}$$

- Carrier-to-Noise Ratio at Ground Rx LNA Input

$$\left(\frac{C}{N_o}\right)_{i,Hub} = \frac{G_i S_i}{G_i N_{o,up,i} + im_o + \frac{N_{o,i,Hub}}{G_{D,i}}}$$

Necessary Link Closure Conditions

- Required Bit-Energy-to-Noise Ratios & Bit Rates

$$\left(\frac{C}{N_o}\right)_i = \left(\frac{E_b}{N_o}\right)_o R_{b,i}, \qquad i = 1,\ldots,N$$

- Received Carrier-to-Noise Ratio

$$\left(\frac{C}{N_o}\right)_{i,Hub} \equiv \left(\frac{C}{N_o}\right)_i = \frac{G_i S_i}{G_i N_{o,up,i} + f_i}; \qquad f_i = i m_o + \frac{N_{o,i,Hub}}{G_{D,i}}$$

- Desired Uplink Signal Powers

$$S_i = N_{o,up,i}\left(\frac{C}{N_o}\right)_i + \frac{w_i}{G_i}; \qquad w_i = \left(\frac{C}{N_o}\right)_i f_i$$

- Link Closure Conditions

$$S_i > N_{o,up,i}\left(\frac{C}{N_o}\right)_i$$

Demands of Transponder Powers

$$Z_i(t) = G_i(t)\left[S_i(t) + N_{o,up,i}(t)\,BW_i\right], \qquad i = 1,\dots,N$$

$$= G_i(t)\,N_{o,up,i}(t)\left[\left(\frac{C}{N_o}\right)_i + BW_i\right] + w_i(t)$$

- Demand and Supply Disequilibrium

$$\frac{d}{dt}Y_i(t) = \alpha_i\left[Z_i(t) - Y_i(t)\right]; \qquad \alpha_i > 0$$

- Asymptotically Stable Dynamics

$$\frac{d}{dt}Y_i(t) = -\alpha_i Y_i(t) + \alpha_i N_{o,up,i}(t)\left[\left(\frac{C}{N_o}\right)_i + BW_i\right]G_i(t) + \alpha_i w_i(t)$$

An Application to Stabilization

$$J_i\left(G_i(\cdot)\right) = Q_i^f\left[Y_i(t_f) - \gamma_i(t_f)\right]^2 + \int_{t_0}^{t_f}\left\{Q_i(t)\left[Y_i(t) - \gamma_i(t)\right]^2 + R_i(t)\left[G_i(t) - \rho_i(t)\right]^2\right\}dt; \qquad i = 1, \ldots, N$$

- Target Transponder Output Powers

$$\gamma_i(t) = P_{\max}\frac{BW_i}{BW_T}$$

- Desired Sub-Channel Gains

$$\rho_i(t) = \frac{w_i(t)}{S_{i,\max} - N_{o,up,i}(t)\left(\frac{C}{N_o}\right)_i}$$

Policy for Sub-Channel Gains

$$G_i(t) = K_i(t) Y_i(t) + l_i(t) + \rho_i(t); \qquad i = 1, \ldots, N$$

- Disequilibrium Dynamics

$$dx_i(t) = \left(A_i(t) + B_i(t) K_i(t)\right) x_i(t)\,dt + B_i(t)\left(l_i(t) + \rho_i(t)\right) dt + E_i(t)\,dw_i(t)$$

where

$$A_i(t) = -\alpha_i; \qquad E_i(t) = \alpha_i; \qquad x_i(t) = Y_i(t)$$

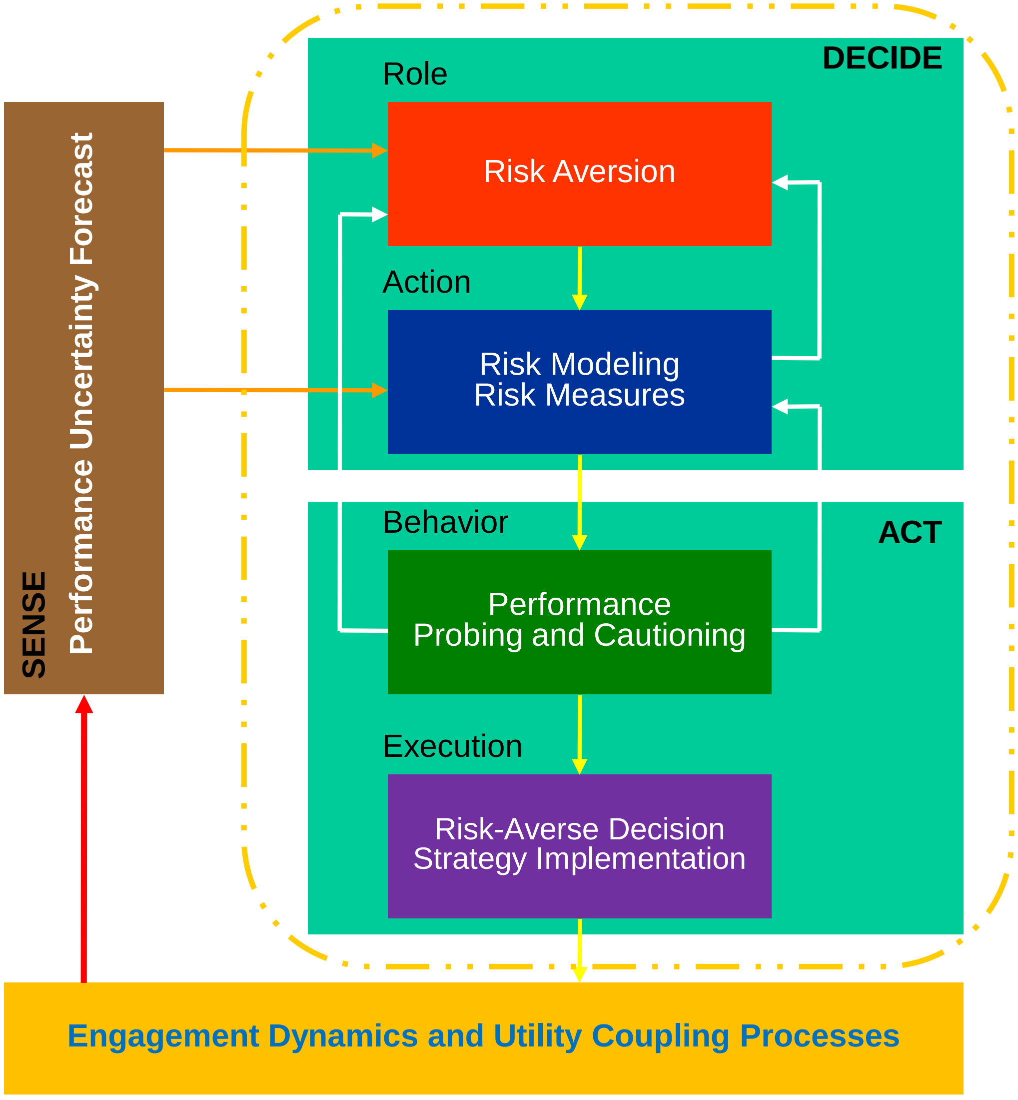
$$B_i(t) = \alpha_i N_{o,up,i}(t) \left[\left(\frac{C}{N_o} \right)_i + BW_i \right]$$

- Realized Performance Measures

$$J_i\left(x_i(t_0); K_i(\cdot), l_i(\cdot)\right) = Q_i^f \left[x_i(t_f) - \gamma_i(t_f) \right]^2 + \int_{t_0}^{t_f} \left\{ Q_i(t) \left[x_i(t) - \gamma_i(t) \right]^2 + R_i(t) \left[K_i(t) x_i(t) + l_i(t) \right]^2 \right\} dt$$

Alerting for mitigation of performance risk and uncertainty

Risk-Averse Decision Framework



Performance Uncertainty Forecast

- Moment and Cumulant-Generating Functions

$$\varphi_i(\tau, x_{i,\tau};\theta) = \varsigma_i(\tau;\theta) e^{\{\Upsilon_i(\tau;\theta)x_{i,\tau}^2 + 2\eta_i(\tau;\theta)x_{i,\tau}\}}; \qquad \theta > 0$$

$$\psi_i(\tau, x_{i,\tau};\theta) = \Upsilon_i(\tau;\theta)x_{i,\tau}^2 + 2\eta_{i,\tau}(\tau;\theta)x_{i,\tau} + \upsilon_i(\tau;\theta); \qquad i = 1,\ldots,N$$

where

$$\frac{d}{d\tau}\Upsilon_i(\tau;\theta) = -[A_i(\tau) + B_i(\tau)K_i(\tau)]^T\Upsilon_i(\tau;\theta) - \Upsilon_i(\tau;\theta)[A_i(\tau) + B_i(\tau)K_i(\tau)]$$
$$-2\Upsilon_i(\tau;\theta)E_i(\tau)W_iE_i^T(\tau)\Upsilon_i(\tau;\theta) - \theta[Q_i(\tau) + K_i^T(\tau)R_i(\tau)K_i(\tau)]; \qquad \Upsilon_i(t_f;\theta) = \theta Q_i^f$$

$$\frac{d}{d\tau}\eta_i(\tau;\theta) = -[A_i(\tau) + B_i(\tau)K_i(\tau)]^T\eta_i(\tau;\theta) - \Upsilon_i(\tau;\theta)B_i(\tau)[l_i(\tau) + \rho_i(\tau)]$$
$$-\theta[K_i^T(\tau)R_i(\tau)l_i(\tau) - Q_i(\tau)\gamma_i(\tau)]; \qquad \eta_i(t_f;\theta) = -\theta Q_i^f\gamma_i(t_f)$$

$$\frac{d}{d\tau}\upsilon_i(\tau;\theta) = -Tr\{\Upsilon_i(\tau;\theta)E_i(\tau)W_iE_i^T(\tau)\} - 2\eta_i^T(\tau;\theta)B_i(\tau)[l_i(\tau) + \rho_i(\tau)]$$
$$-\theta[l_i^T(\tau)R_i(\tau)l_i(\tau) + \gamma_i^T(\tau)Q_i(\tau)\gamma_i(\tau)]; \qquad \upsilon_i(t_f;\theta) = \theta\gamma_i^T(t_f)Q_i^f\gamma_i(t_f)$$

Risk Modeling and Measures

$$\kappa_r^i\left(\tau, x_{i,\tau}\right) = \frac{\partial^{(r)}}{\partial\theta^{(r)}}\Upsilon_i\left(\tau;\theta\right)|_{\theta=0}\ x_{i,\tau}^2 + 2\frac{\partial^{(r)}}{\partial\theta^{(r)}}\eta_{i,\tau}\left(\tau;\theta\right)|_{\theta=0}\ x_{i,\tau} + \frac{\partial^{(r)}}{\partial\theta^{(r)}}\upsilon_i\left(\tau;\theta\right)|_{\theta=0}$$

$$\kappa_k^i = H_k^i\left(t_0\right)x_i^2\left(t_0\right) + 2\breve{D}_k^i\left(t_0\right)x_i\left(t_0\right) + D_k^i\left(t_0\right); \qquad i = 1,\ldots,N$$

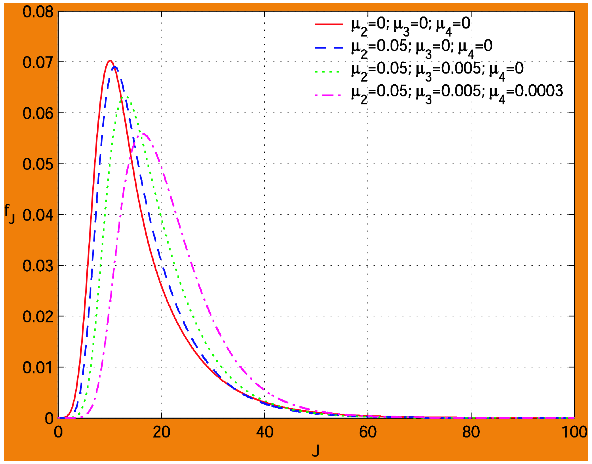
- Cumulants address a host of performance uncertainties

$$\text{Mean: } \kappa_1^i = E\left\{J_i\right\}$$

$$\text{Variance: } \kappa_2^i = E\left\{J_i^2\right\} - E^2\left\{J_i\right\}$$

$$\text{Skewness: } \kappa_3^i = E\left\{J_i^3\right\} - 3E\left\{J_i^2\right\}E\left\{J_i\right\} - 2E^3\left\{J_i\right\}$$

$$\text{Flatness: } \kappa_4^i = E\left\{J_i^4\right\} - 4E\left\{J_i^3\right\}E\left\{J_i\right\} - 3E^2\left\{J_i^2\right\} + 12E\left\{J_i^2\right\}E^2\left\{J_i\right\} - 6E^4\left\{J_i\right\}$$



Variance, Skewness, Kurtosis... as higher-order stochastic dominance

Performance Uncertainty Probing and Cautioning

$$\frac{d}{d\tau}H_1^i(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]^T H_1^i(\tau) - H_1^i(\tau)\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]$$
$$-\left[Q_i(\tau)+K_i^T(\tau)R_i(\tau)K_i(\tau)\right]; \qquad H_1^i(t_f) = Q_i^f$$

$$\frac{d}{d\tau}H_r^i(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]^T H_r^i(\tau) - H_r^i(\tau)\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]$$
$$-\sum_{s=1}^{r-1}\frac{2r!}{s!(r-s)!}H_s^i(\tau)E_i(\tau)W_iE_i^T(\tau)H_{r-s}^i(\tau); \qquad H_r^i(t_f) = 0; \qquad 2 \le r \le k$$

$$\frac{d}{d\tau}\breve{D}_1^i(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]^T \breve{D}_1^i(\tau) - H_1^i(\tau)B_i(\tau)\left[l_i(\tau)+\rho_i(\tau)\right]$$
$$-\left[K_i^T(\tau)R_i(\tau)l_i(\tau)-Q_i(\tau)\gamma_i(\tau)\right]; \qquad \breve{D}_1^i(t_f) = -Q_i^f\gamma_i(t_f)$$

$$\frac{d}{d\tau}\breve{D}_r^i(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i(\tau)\right]^T \breve{D}_r^i(\tau) - H_r^i(\tau)B_i(\tau)\left[l_i(\tau)+\rho_i(\tau)\right]; \quad \breve{D}_r^i(t_f) = 0$$

$$\frac{d}{d\tau}D_1^i(\tau) = -Tr\left\{H_1^i(\tau)E_i(\tau)W_iE_i^T(\tau)\right\} - 2\breve{D}_1^i(\tau)B_i(\tau)\left[l_i(\tau)+\rho_i(\tau)\right]$$
$$-\left[l_i^T(\tau)R_i(\tau)l_i(\tau)+\gamma_i^T(\tau)Q_i(\tau)\gamma_i(\tau)\right]; \qquad D_1^i(t_f) = \gamma_i^T(t_f)Q_i^f\gamma_i(t_f)$$

$$\frac{d}{d\tau}D_r^i(\tau) = -Tr\left\{H_r^i(\tau)E_i(\tau)W_iE_i^T(\tau)\right\} - 2\breve{D}_r^i(\tau)B_i(\tau)\left[l_i(\tau)+\rho_i(\tau)\right]; \quad D_r^i(t_f) = 0$$

Performance Uncertainty Probing and Cautioning

Let the product mappings be defined by

$$F^i = F_1^i \times \cdots \times F_k^i; \qquad \breve{G}^i = \breve{G}_1^i \times \cdots \times \breve{G}_k^i; \qquad G^i = G_1^i \times \cdots \times G_k^i$$

Then

$$\frac{d}{d\tau} H^i(\tau) = F^i\big(\tau, H^i(\tau), K_i(\tau)\big), \qquad H^i(t_f) \equiv H_f^i$$

$$\frac{d}{d\tau} \breve{D}^i(\tau) = \breve{G}^i\big(\tau, H^i(\tau), K_i(\tau), l_i(\tau)\big), \qquad \breve{D}^i(t_f) \equiv \breve{D}_f^i$$

$$\frac{d}{d\tau} D^i(\tau) = G^i\big(\tau, H^i(\tau), l_i(\tau)\big), \qquad D^i(t_f) \equiv D_f^i$$

and

$$H_f^i = \Big(Q_f^i, \underbrace{0, \cdots, 0}_{(k-1)\text{-times}}\Big); \quad \breve{D}_f^i = \Big(-Q_i^f \gamma_i(t_f), \underbrace{0, \cdots, 0}_{(k-1)\text{-times}}\Big); \quad D_f^i = \Big(\gamma_i^T(t_f) Q_i^f \gamma_i(t_f), \underbrace{0, \cdots, 0}_{(k-1)\text{-times}}\Big)$$

Mean-Risk Aware Performance Index

A "correct-by-design" approach to finding a solution that best satisfies performance reliability

$$\phi_0^i\left(K_i, l_i\right) = \underbrace{\mu_1^i \kappa_1^i}_{\text{Mean Measure}} + \underbrace{\mu_2^i \kappa_2^i + \cdots + \mu_k^i \kappa_{k^i}^i}_{\text{Risk Measures}}$$

Different tradeoffs on the mathematical statistics against performance risks are being weighted through the risk-averse profiles

$$\mu^i = \left\{\mu_r^i \geq 0\right\}_{r=1}^{k} \text{ with } \mu_1^i > 0.$$

Preferences being modeled by dynamic measures of risks

Statistical Optimal Control Problem

For any given risk-averse profile, the statistical optimal control optimization is described as follows:

$$\min_{(K_i, l_i) \in \mathcal{K}^i_{t_f, H^i_f, \breve{D}^i_f, D^i_f; \mu^i} \times \mathcal{L}^i_{t_f, H^i_f, \breve{D}^i_f, D^i_f; \mu^i}} \phi^i_0(K_i, l_i)$$

subject to the dynamical equations

$$\frac{d}{d\tau} H^i(\tau) = F^i\left(\tau, H^i(\tau), K_i(\tau)\right); \qquad H^i(t_f) \equiv H^i_f$$

$$\frac{d}{d\tau} \breve{D}^i(\tau) = \breve{G}^i\left(\tau, H^i(\tau), K_i(\tau), l_i(\tau)\right); \qquad \breve{D}^i(t_f) \equiv \breve{D}^i_f$$

$$\frac{d}{d\tau} D^i(\tau) = G^i\left(\tau, H^i(\tau), l_i(\tau)\right); \qquad D^i(t_f) \equiv D^i_f$$

Risk-Averse Sub-Channel Gains

$$G_i^*(t) = K_i^*(t)x_i^*(t) + l_i^*(t) + \rho_i(t); \qquad t = t_f - \tau$$

$$K_i^*(\tau) = -R_i^{-1}(\tau)B_i^T(\tau)\sum_{r=1}^{k}\hat{\mu}_r^i H_r^{i*}(\tau);$$

$$l_i^*(\tau) = -R_i^{-1}(\tau)B_i^T(\tau)\sum_{r=1}^{k}\hat{\mu}_r^i \breve{D}_r^{i*}(\tau); \qquad \hat{\mu}_r^i = \frac{\mu_r^i}{\mu_1^i}$$

Assessing and cross-checking sub-channel transponder output powers for the demand and supply disequilibrium

$$dx_i^*(t) = \left(A_i(t) + B_i(t)K_i^*(t)\right)x_i^*(t)dt + B_i(t)\left(l_i^*(t) + \rho_i(t)\right)dt + E_i(t)dw_i(t); \quad x_i^*(t_0) = x_{i0}$$

Risk-Averse Sub-Channel Gains

Integrating performance uncertainty forecast for performance reliability considerations

$$\frac{d}{d\tau}H_1^{i*}(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right]^T H_1^{i*}(\tau) - H_1^{i*}(\tau)\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right] - \left[Q_i(\tau)+K_i^{*T}(\tau)R_i(\tau)K_i^*(\tau)\right]; \qquad H_1^{i*}(t_f) = Q_i^f$$

$$\frac{d}{d\tau}H_r^{i*}(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right]^T H_r^{i*}(\tau) - H_r^{i*}(\tau)\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right] - \sum_{s=1}^{r-1}\frac{2r!}{s!(r-s)!}H_s^{i*}(\tau)E_i(\tau)W_iE_i^T(\tau)H_{r-s}^{i*}(\tau); \qquad H_r^{i*}(t_f)=0; \qquad 2\le r\le k$$

$$\frac{d}{d\tau}\breve{D}_1^{i*}(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right]^T \breve{D}_1^{i*}(\tau) - H_1^{i*}(\tau)B_i(\tau)\left[l_i^*(\tau)+\rho_i(\tau)\right] - \left[K_i^{*T}(\tau)R_i(\tau)l_i^*(\tau)-Q_i(\tau)\gamma_i(\tau)\right]; \qquad \breve{D}_1^{i*}(t_f) = -Q_i^f\gamma_i(t_f)$$

$$\frac{d}{d\tau}\breve{D}_r^{i*}(\tau) = -\left[A_i(\tau)+B_i(\tau)K_i^*(\tau)\right]^T \breve{D}_r^{i*}(\tau) - H_r^{i*}(\tau)B_i(\tau)\left[l_i^*(\tau)+\rho_i(\tau)\right]$$

$$\breve{D}_r^{i*}(t_f)=0; \qquad 2\le r\le k$$

Design Principles / Implementation

$$\frac{d}{d\tau} H^{i*}(\tau) = F^i\left(\tau, H^{i*}(\tau), K_i^*(\tau)\right)$$

$$K_i^*(t) = -R_i^{-1}(t) B_i^T(t) \sum_{r=1}^{k} \hat{\mu}_r^i H_r^{i*}(t)$$

$$\frac{d}{d\tau} \breve{D}^{i*}(\tau) = \breve{G}^i\left(\tau, H^{i*}(\tau), \breve{D}^{i*}(\tau), K_i^*(\tau), l_i^*(\tau)\right)$$

$$l_i^*(t) = -R_i^{-1}(t) B_i^T(t) \sum_{r=1}^{k} \hat{\mu}_r^i \breve{D}_r^{i*}(t)$$

$K_i^*(t)$

$\eta_{K_i^*} = \left(t, x_i^*(t)\right)$

$l_i^*(t)$

$\eta_{l_i^*} = \left(t, \gamma_i(t), \rho_i(t)\right)$

$$dx_i^*(t) = \left(A_i(t) x_i^*(t) + B_i(t) G_i^*(t)\right) dt + E_i(t) dw_i(t); \qquad x_i^*(t_0) = x_{i0}; \qquad t \in [t_0, t_f]$$

Desired Transponder Output Power $\gamma_i(t)$ and Reference Sub-Channel Gain $\rho_i(t)$

$$G_i^*(t) = K_i^*(t) x_i^*(t) + l_i^*(t) + \rho_i(t)$$

$$J_i(t_0, x_{i0}) = \left[x_i(t_f) - \gamma_i(t_f)\right]^T Q_i^f \left[x_i(t_f) - \gamma_i(t_f)\right]$$

$$+ \int_{t_0}^{t_f} \left\{ \left[x_i^*(\tau) - \gamma_i(\tau)\right]^T Q_i(\tau) \left[x_i^*(\tau) - \gamma_i(\tau)\right] + \left[G_i^*(\tau) - \rho_i(\tau)\right]^T R_i(\tau) \left[G_i^*(\tau) - \rho_i(\tau)\right] \right\} d\tau$$

Uplink Full-Band Full-Time Jamming

- Average Receive Jamming Power

$$J_{jammer} = \frac{EIRP_{jammer} G_{PL,Rx}}{L_{u,path} L_{u,rain} L_{u,misc}}$$

- Jammer Noise Density

$$J_0 = \frac{J_{jammer}}{BW_T}$$

- Downlink Carrier-to-Noise Ratios

$$\left(\frac{C}{N_o}\right)_{i,Hub} = \frac{G_i S_i}{G_i \left[N_{o,up,i} + J_0 \right] + im_o + \frac{N_{o,i,Hub}}{G_{D,i}}}$$

Multi-Link Powers Demanded from Transponder

$$Z_i(t) = G_i(t)\left[N_{o,up,i}(t) + J_0\right]\left[\left(\frac{C}{N_o}\right)_i + BW_i\right] + w_i(t); \qquad i = 1,\ldots,N$$

- Demand and Supply Disequilibrium

$$\frac{d}{dt}Y_i(t) = -\alpha_i Y_i(t) + \alpha_i\left[N_{o,up,i}(t) + J_0\right]\left[\left(\frac{C}{N_o}\right)_i + BW_i\right]G_i(t) + \alpha_i w_i(t)$$

- Multi-Link Powers Demanded from the Transponder

$$\frac{d}{dt}Y(t) = A(t)Y(t) + B(t)U(t) + E(t)W(t); \qquad Y(t_0)$$

$$Y(t) = \left[Y_1(t) \quad Y_2(t) \quad \cdots \quad Y_N(t)\right]^T \qquad A(t) = diag\left(A_1(t), A_2(t), \ldots, A_N(t)\right)$$

$$U(t) = \left[G_1(t) \quad G_2(t) \quad \cdots \quad G_N(t)\right]^T \qquad B(t) = diag\left(B_1(t), B_2(t), \ldots, B_N(t)\right)$$

$$W(t) = \left[w_1(t) \quad w_2(t) \quad \cdots \quad w_N(t)\right]^T \qquad E(t) = diag\left(E_1(t), E_2(t), \ldots, E_N(t)\right)$$

Performance Measure of Multi-Link Power Allocation

$$J\left(U(\cdot)\right)=\left[Y(t_f)-Y_r(t_f)\right]^T Q^f\left[Y(t_f)-Y_r(t_f)\right]$$
$$+\int_{t_0}^{t_f}\left\{\left[Y(t)-Y_r(t)\right]^T Q(t)\left[Y(t)-Y_r(t)\right]+\left[U(t)-U_r(t)\right]^T R(t)\left[U(t)-U_r(t)\right]\right\}dt$$

where

$$Y_r(t)=\begin{bmatrix} P_{\max}\dfrac{BW_1}{BW_T} & P_{\max}\dfrac{BW_2}{BW_T} & \cdots & P_{\max}\dfrac{BW_N}{BW_T}\end{bmatrix}$$

$$U_r(t)=\begin{bmatrix} \dfrac{w_1}{S_{1,\max}-\left[N_{o,up,1}(t)+J_0\right]\left(\dfrac{C}{N_o}\right)_1} & \dfrac{w_2}{S_{2,\max}-\left[N_{o,up,2}(t)+J_0\right]\left(\dfrac{C}{N_o}\right)_2} & \cdots & \dfrac{w_N}{S_{N,\max}-\left[N_{o,up,N}(t)+J_0\right]\left(\dfrac{C}{N_o}\right)_N}\end{bmatrix}$$

Risk-Averse Multi-Link Sub-Channel Gains

$$U^*(t) = K^*(t)Y^*(t) + l^*(t) + U_r(t); \qquad t = t_f - \tau$$

$$K^*(\tau) = -R^{-1}(\tau)B^T(\tau)\sum_{r=1}^{k}\hat{\mu}_r H_r^*(\tau);$$

$$l^*(\tau) = -R^{-1}(\tau)B^T(\tau)\sum_{r=1}^{k}\hat{\mu}_r \breve{D}_r^*(\tau); \qquad \hat{\mu}_r = \frac{\mu_r}{\mu_1}$$

subject to sub-channel transponder output powers for potential demand and supply disequilibrium

$$\frac{d}{dt}Y^*(t) = A(t)Y^*(t) + B(t)U^*(t) + E(t)W(t); \qquad Y^*(t_0)$$

Risk-Averse Multi-Link Sub-Channel Gains

robust against random sample-path realization surprises

$$\frac{d}{d\tau}H_1^*(\tau) = -\left[A(\tau)+B(\tau)K^*(\tau)\right]^T H_1^*(\tau) - H_1^*(\tau)\left[A(\tau)+B(\tau)K^*(\tau)\right] - \left[Q_i(\tau)+K^{*T}(\tau)R(\tau)K^*(\tau)\right]; \qquad H_1^*(t_f) = Q^f$$

$$\frac{d}{d\tau}H_r^*(\tau) = -\left[A(\tau)+B(\tau)K^*(\tau)\right]^T H_r^*(\tau) - H_r^*(\tau)\left[A(\tau)+B(\tau)K^*(\tau)\right] - \sum_{s=1}^{r-1}\frac{2r!}{s!(r-s)!}H_s^*(\tau)E(\tau)WE^T(\tau)H_{r-s}^*(\tau); \qquad H_r^*(t_f) = 0; \qquad 2 \le r \le k$$

$$\frac{d}{d\tau}\breve{D}_1^*(\tau) = -\left[A(\tau)+B(\tau)K^*(\tau)\right]^T \breve{D}_1^*(\tau) - H_1^*(\tau)B(\tau)\left[l^*(\tau)+U_r(\tau)\right] - \left[K^{*T}(\tau)R(\tau)l^*(\tau) - Q(\tau)Y_r(\tau)\right]; \qquad \breve{D}_1^*(t_f) = -Q^f Y_r(t_f)$$

$$\frac{d}{d\tau}\breve{D}_r^*(\tau) = -\left[A(\tau)+B(\tau)K^*(\tau)\right]^T \breve{D}_r^*(\tau) - H_r^*(\tau)B(\tau)\left[l^*(\tau)+U_r(\tau)\right]$$

$$\breve{D}_r^*(t_f) = 0; \qquad 2 \le r \le k$$

Conclusions

- Statistical optimal control approach to power allocation shared satellite transponders
  - Nonlinear multi-mode carrier transponders
  - Selective sub-channel gains
  - Full-band full-time noise jamming
- Risk-averse power allocation enabled by
  - Performance uncertainty forecast and management
  - Risk aversion hedging via "correct-by-construction" for desired carrier-to-noise ratios at ground hubs
- Future work involving
  - User-case simulation analysis

Questions?