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Resilient Synchronization of Remote Time Dissemination

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Khanh Pham, PhD
Space Vehicles Directorate
Air Force Research Laboratory

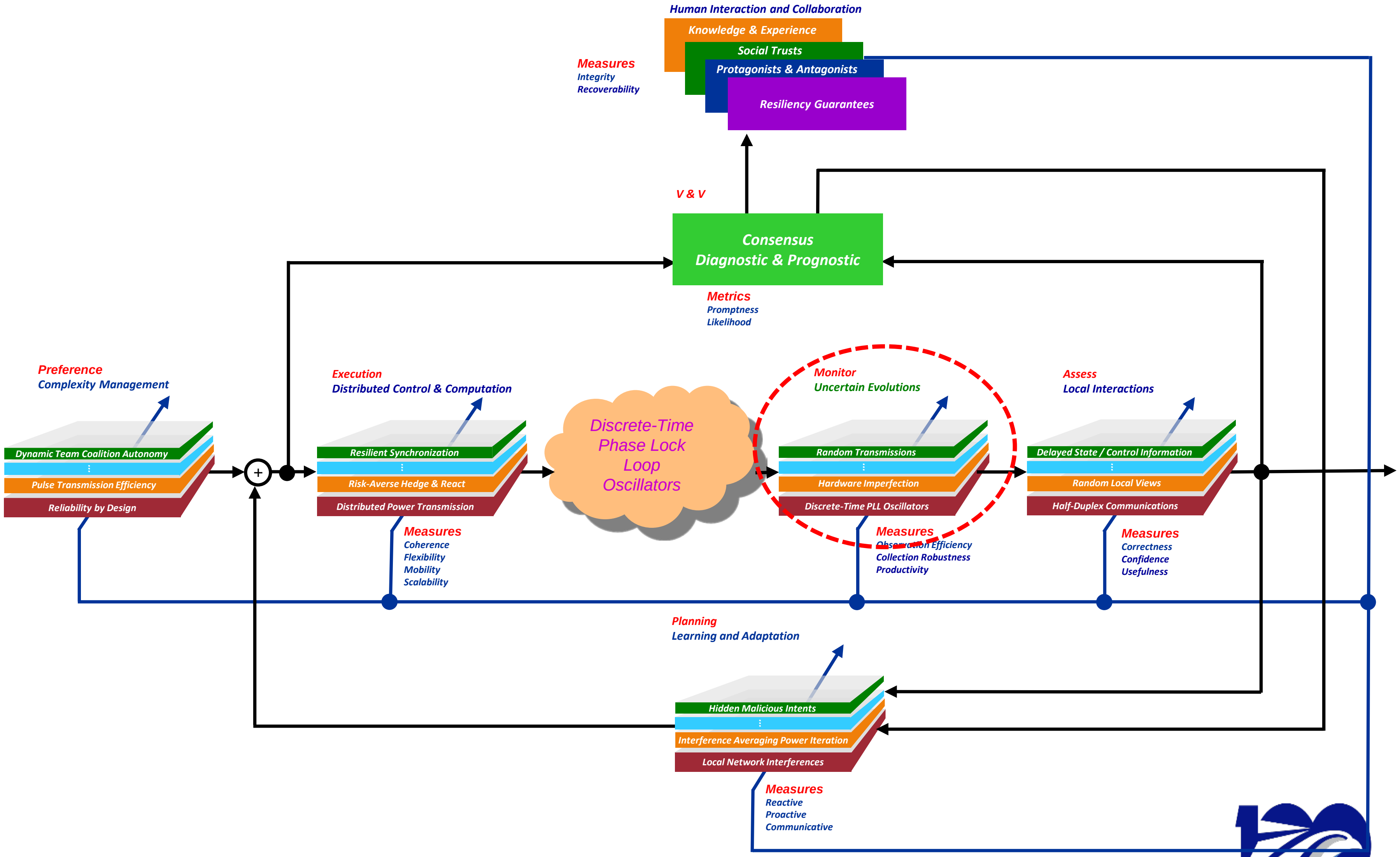
Integrity ★ Service ★ Excellence

Expected Contributions

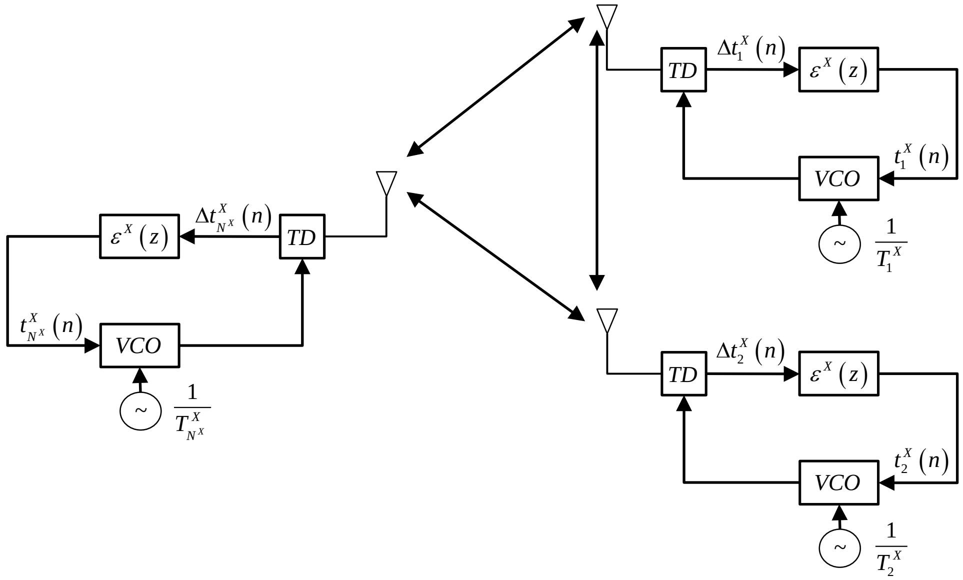
- **Distributed synchronization in wireless communications subject to**
  - **Diversity of threats; e.g., Spread of misinformation**
- **Focus on a multi-agent graphical game framework for physical-layer based time synchronization**
  - **Coupled discrete-time oscillators**
  - **Disagreements in timing information among protagonist and antagonist clocks with diversity reception**
- **Adaptive pulse-coupled synchronization**
  - **Resiliency via pursuit-evasion graphical games**

Time Distribution, Synchronization of Clocks, etc.

Resilient Time Synchronization



Monitor
Pulse-Coupled Discrete-Time Clocks



- **Clock Communications Enabled by Sequence of Weighted Graphs**

$$\mathcal{G}^X(n) = \left(\mathcal{V}^X, \mathcal{A}^X(n), w^X(n)\right); \qquad n \in \mathbb{N}; \qquad X = \{P, E\}$$

$$\mathcal{V}^X = \{1, \ldots, N^X\}$$

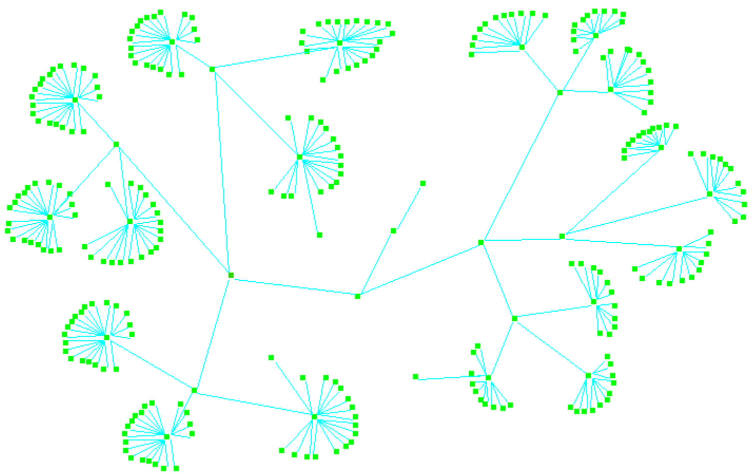
$$w^X : \mathcal{A}^X \times \mathbb{N} \mapsto \mathbb{R}_+$$

Monitor
Pulse-Coupled Discrete-Time Clocks

- **Random Transmits by Bernoulli RVs** $m_k^X \in \{0,1\}$ and $X = \{P, E\}$

$$I_k^X(n) = \begin{cases} 0 & \text{Tx with } 1-\beta_k^X = \Pr\left[m_k^X = 0\right] \\ 1 & \text{Rx with } \quad \beta_k^X = \Pr\left[m_k^X = 1\right] \end{cases}$$

$$\overline{I}_k^X(n) = \begin{cases} 1 & \text{if } I_k^X(n) = 0 \\ 0 & \text{if } I_k^X(n) = 1 \end{cases}$$

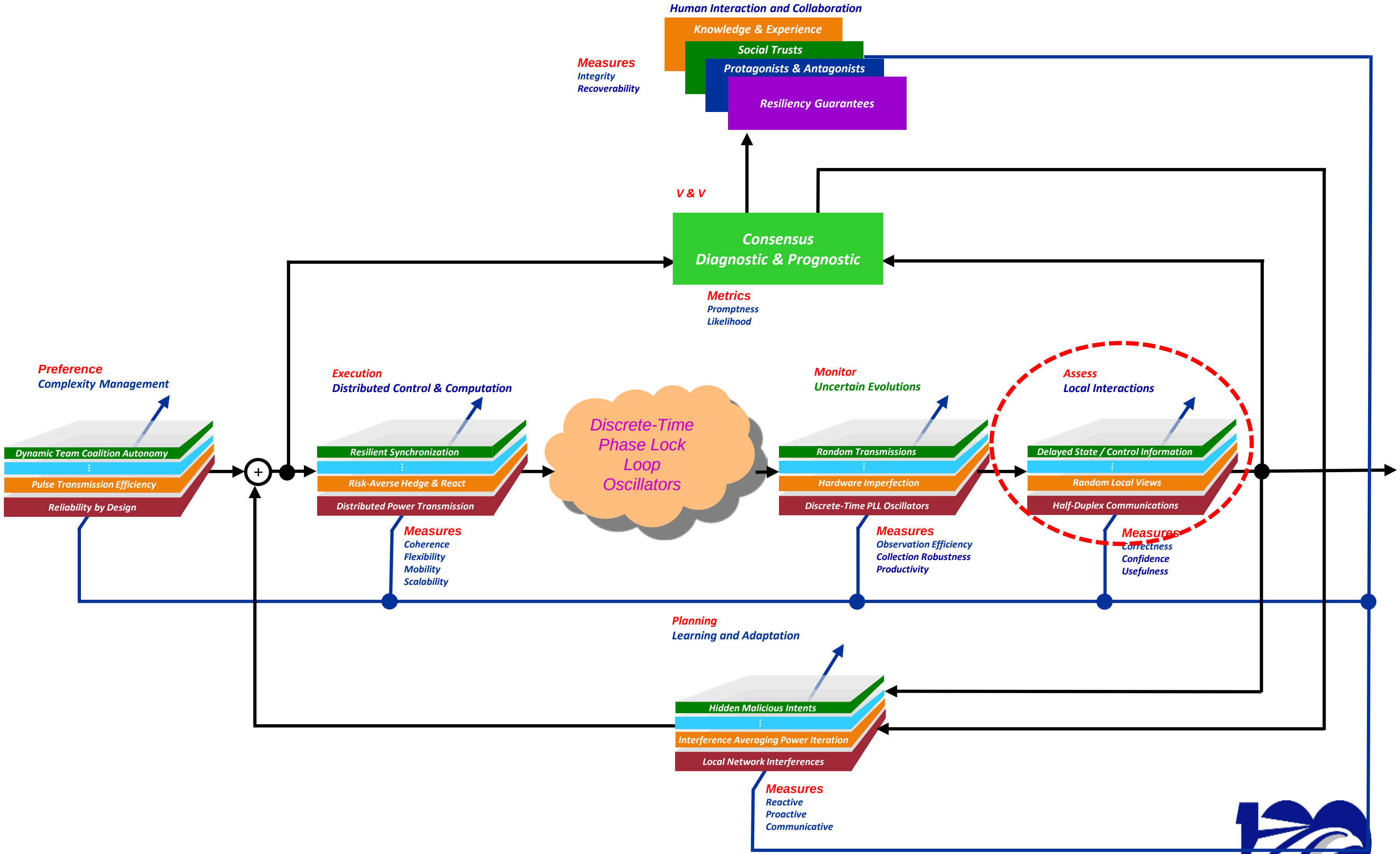


- **Local Views**

$$t_k^X(n+1) = t_k^X(n) + \varepsilon^X \Delta t_k^X(n+1) + T_k^X + v_k^X(n); \qquad n \in \mathbb{N}; \qquad X = \{P, E\}$$

$$\Delta t_k^X(n+1) = \frac{I_k^X(n)}{\left|\text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right)\right|} \sum_{i \in \text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right)} \overline{I}_i^X(n) w_{ki}^X(n)\left[t_i^X(n) - t_k^X(n)\right]$$

Resilient Time Synchronization



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Access
Local Interactions

- **Local Coupling Coefficients**

$$w_{ki}^{X}(n) = \frac{p_{ki}^{X}(n)}{\sum_{i \in \text{Neighbors}\left(\{k\}, \mathcal{A}^{X}(n)\right)} p_{ki}^{X}(n)}; \qquad n \in \mathbb{N}$$

$$w_{ki}^{X}(n) > 0 \text{ and } \sum_{i \in \text{Neighbors}\left(\{k\}, \mathcal{A}^{X}(n)\right)} w_{ki}^{X}(n) = 1$$

- **Frequency Synchrony:** $t_{k}^{X}(n) = nT^{X} + \tau_{k}^{X}(n)$ and $T^{X} = T_{1}^{X} = \cdots = T_{N^{X}}^{X}$

$$\tau_{k}^{X}(n+1) = \tau_{k}^{X}(n) + \varepsilon^{X} \Delta\tau_{k}^{X}(n+1) + v_{k}^{X}(n); \qquad n \in \mathbb{N}; \qquad 0 < \varepsilon^{X} < 1$$

$$\Delta\tau_{k}^{X}(n+1) = \frac{I_{k}^{X}(n)}{\left|\text{Neighbors}\left(\{k\}, \mathcal{A}^{X}(n)\right)\right|} \sum_{i \in \text{Neighbors}\left(\{k\}, \mathcal{A}^{X}(n)\right)} \overline{I}_{i}^{X}(n) w_{ki}^{X}(n) \left[\tau_{i}^{X}(n) - \tau_{k}^{X}(n)\right]$$

Access
Local Interactions

- **Local Dynamics and Inputs of Disagreements**

$$x_k^X(n+1) = x_k^X(n) + \varepsilon^X u_k^X(n) + v_k^X(n); \qquad n \in \mathbb{N}; \qquad X = \{P, E\}$$

$$u_k^X(n) = \frac{I_k^X(n)}{\left|\text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right)\right|} \sum_{i \in \text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right)} \overline{I}_i^X(n) w_{ki}^X(n) \left[x_i^X(n) - x_k^X(n)\right]$$

- **Complete Information**

$$\left\{x_i^X(n) - x_k^X(n), u_i^X(n), u_k^X(n)\right\}$$

Setting Directions for Processing

Access
Local Interactions

- **Let**

$$\mathcal{N}_k^X = \text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right) \cup \{k\}; \qquad \left|\mathcal{N}_k^X\right| = N_k^X; \qquad X = \{P, E\}$$

$$x_{\mathcal{N}_k^X}^X(n) = \left[x_1^X(n), \ldots, x_{N_k^X}^X(n)\right]; \qquad v_{\mathcal{N}_k^X}^X(n) = \left[v_1^X(n), \ldots, v_{N_k^X}^X(n)\right]$$

- **Cluster Dynamics of Pulse Coupled Discrete Time Synchronization**

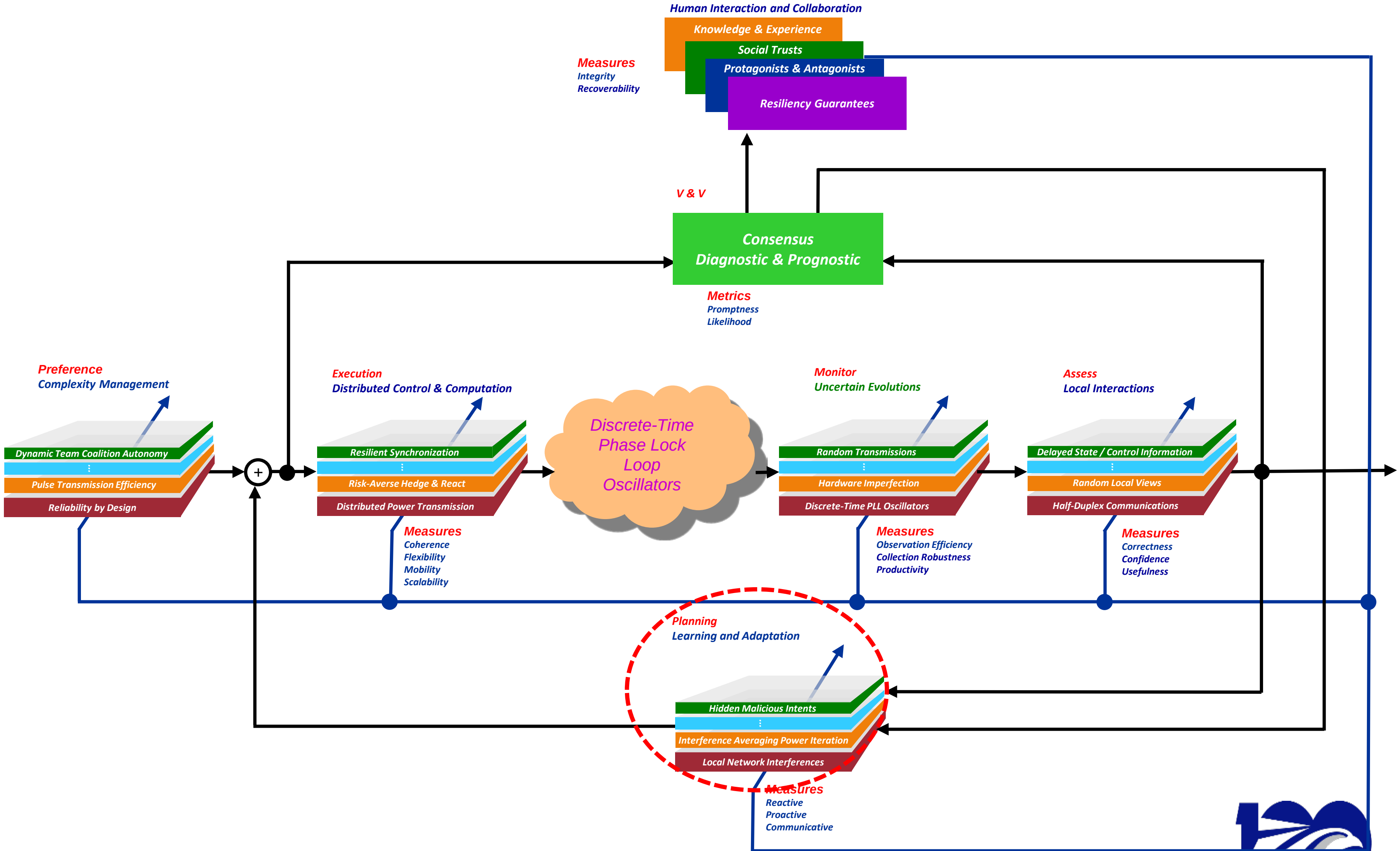
$$x_{\mathcal{N}_k^X}^X(n+1) = A_{\mathcal{N}_k^X}^X(n)\, x_{\mathcal{N}_k^X}^X(n) + v_{\mathcal{N}_k^X}^X(n); \qquad X = \{P, E\}$$

where

$$\left[A_{\mathcal{N}_k^X}^X(n)\right]_{ii} = 1 - \varepsilon^X$$

$$\left[A_{\mathcal{N}_k^X}^X(n)\right]_{ki} = \varepsilon^X \frac{I_k^X(n)}{\left|\text{Neighbors}\left(\{k\}, \mathcal{A}^X(n)\right)\right|} \bar{I}_i^X(n)\, w_{ki}^X(n)$$

Resilient Time Synchronization



Planning
Learning and Adaptation

- **Pulse Signal Power of Clock i Received by Clock k**

$$h_{ki}^{X}(n)\, p_i^{X}(n); \qquad n \in \mathbb{N}; \qquad X = \{P, E\}$$

- **Local Interferences Seen by Clock i at Clock k**

$$\sum_{j \in \mathcal{N}_k^X} h_{kj}^{X}(n)\, p_j^{X}(n) + \sigma_k^{X}(n)$$

- **Define**

$$p_{\mathcal{N}_k^X}^{X}(n) = \left[p_1^{X}(n), \ldots, p_{N_k^X}^{X}(n) \right]$$

- **Signal-to-Interference-plus-Noise Ratio (SINR) of Clock i at Clock k; i.e.,** $p_i^{X}(n)\, \mu_{ki}^{X}(n)$

$$\mu_{ki}^{X}\left(p_{\mathcal{N}_k^X}\right) = \frac{h_{ki}^{X}(n)}{\sum_{j \in \mathcal{N}_k^X \setminus \{i\}} h_{kj}^{X}(h)\, p_j^{X}(n) + \sigma_k^{X}(n)}$$

Planning
Learning and Adaptation

- **Maximal Ratio Combining**

$$p_i^X(n)\sum_{k\in\mathcal{N}_i^X}\mu_{ki}^X\left(p_{\mathcal{N}_i^X}^X\right)\geq\gamma_i^X(n)$$

of the received pulse signals for clock i at its immediate neighbors k

- **Or, equivalently**

$$p_i^X(n)\geq I_i^X\left(p_{\mathcal{N}_i^X}^X\right)=\frac{\gamma_i^X(n)}{\sum_{k\in\mathcal{N}_i^X}\mu_{ki}^X\left(p_{\mathcal{N}_i^X}^X\right)}$$

Positivity: $I_i^X\left(p_{\mathcal{N}_i^X}^X\right)>0$

Monotonicity: $I_i^X\left(p_{\mathcal{N}_i^X}^X\right)\geq I_i^X\left(\tilde{p}_{\mathcal{N}_i^X}^X\right)$ when $p_{\mathcal{N}_i^X}^X\geq\tilde{p}_{\mathcal{N}_i^X}^X$

Scalability: $\alpha I_i^X\left(p_{\mathcal{N}_i^X}^X\right)>I_i^X\left(\alpha p_{\mathcal{N}_i^X}^X\right)$ when $\alpha>1$

- **Interference Averaging Power Control Iteration**

$$p^X_{\mathcal{N}_k^X}(n+1) = \pi^X_{\mathcal{N}_k^X} p^X_{\mathcal{N}_k^X}(n) + \left(1-\pi^X_{\mathcal{N}_k^X}\right) I^X_{\mathcal{N}_k^X}\left(p^X_{\mathcal{N}_k^X}(n)\right); \qquad 0 \le \pi^X_{\mathcal{N}_k^X} < 1; \qquad n \in \mathbb{N}$$

$$I^X_{\mathcal{N}_k^X}\left(p^X_{\mathcal{N}_k^X}(n)\right) = \left[I_1^X\left(p^X_{\mathcal{N}_1^X}(n)\right), \ldots, I^X_{N_k^X}\left(p^X_{\mathcal{N}_k^X}(n)\right)\right]; \; I_i^X\left(p^X_{\mathcal{N}_i^X}(n)\right) = \frac{\gamma_i^X(n)}{\sum_{k \in \mathcal{N}_i^X} \mu^X_{ki}\left(p^X_{\mathcal{N}_i^X}(n)\right)}$$

- **Or, equivalently**

$$p^X_{\mathcal{N}_k^X}(n+1) = A^{X,p}_{\mathcal{N}_k^X}(n)\, p^X_{\mathcal{N}_k^X}(n) + B^{X,p}_{\mathcal{N}_k^X}(n)\, u^X_{\mathcal{N}_k^X}(n); \qquad X = \{P, E\}$$

where

$$A^{X,p}_{\mathcal{N}_k^X}(n) = \pi^X_{\mathcal{N}_k^X} I_{N_k^X \times N_k^X}; \qquad u^X_{\mathcal{N}_k^X}(n) = \left[\gamma_1^X(n), \ldots, \gamma^X_{N_k^X}(n)\right]$$

$$B^{X,p}_{\mathcal{N}_k^X}(n) = \left(1-\pi^X_{\mathcal{N}_k^X}\right) diag\left(\frac{1}{\sum_{k \in \mathcal{N}_1^X} \mu^X_{k1}\left(p^X_{\mathcal{N}_k^X}(n)\right)}, \ldots, \frac{1}{\sum_{k \in \mathcal{N}_{N_k}} \mu^X_{kN_k^X}\left(p^X_{\mathcal{N}_k^X}(n)\right)} \right)$$

Planning
Learning and Adaptation

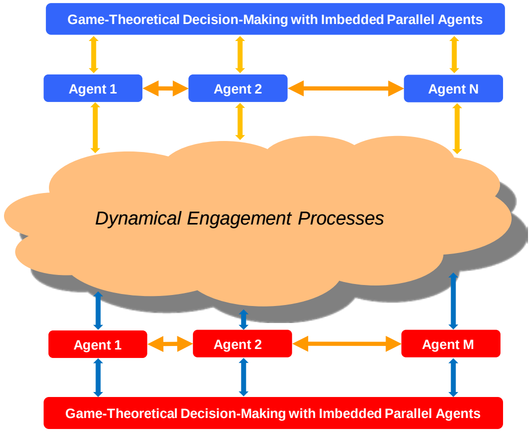
- **Protagonist and Antagonist Interactions**

$$\mathcal{G}^{EP}(n) = \left(\mathcal{V}^{E}, \mathcal{V}^{P}, \mathcal{E}^{EP}, \mathcal{A}^{EP}(n)\right); \qquad n \in \mathbb{N}$$

$$\mathcal{V}^{E} = \left\{1, \ldots, N^{E}\right\}; \qquad \mathcal{V}^{P} = \left\{1, \ldots, N^{P}\right\}; \qquad \mathcal{E}^{EP} \subseteq \mathcal{V}^{E} \times \mathcal{V}^{P}$$

- **Powers Received by Protagonist Clock i Transmitted by Antagonist Clock j**

$$p_{ij}^{EP}(n) = \frac{G_{i,j,Tx}^{P} G_{i,j,Rx}^{E} \lambda_j^2}{\left(d_{i,j}^{EP}(n)\right)^2} p_j^P(n)$$



- **Time-Variant Weighted Adjacent Matrix**

$$\left[A^{EP}(n)\right]_{ij} = \frac{p_{ij}^{EP}(n)}{\sum_{j \in \mathcal{N}_i^{EP}} p_{ij}^{EP}(n)}$$

Planning
Learning and Adaptation

- **Antagonist Clocks Tending to Attract Other Antagonist and Protagonist Clocks**

$$e^P_{x_i^P}(n) = \sum_{k \in \mathcal{N}_i^P} \left[A^P(n)\right]_{ki} \left[x_k^P(n) - x_i^P(n)\right] + \sum_{j \in \mathcal{N}_i^E} \left[A^{EP}(n)\right]_{ji} \left[x_j^E(n) - x_i^P(n)\right]$$

- **Protagonist Clocks Tending to Attract Other Protagonist Clocks But Avoiding Antagonist Clocks**

$$e^E_{x_j^E}(n) = \sum_{l \in \mathcal{N}_j^E} \left[A^E(n)\right]_{jl} \left[x_l^E(n) - x_j^E(n)\right] + \sum_{i \in \mathcal{N}_j^P} \left[A^{EP}(n)\right]_{ji} \left[x_i^P(n) - x_j^E(n)\right]$$

Planning
Learning and Adaptation

$$e^{P}_{x_{\mathcal{N}_i^P}}(n)=\begin{bmatrix} e^{P}_{x_1^P}(n) & \cdots & e^{P}_{x^P_{N_i^P}}(n)\end{bmatrix}; \qquad e^{E}_{x_{\mathcal{N}_j^E}}(n)=\begin{bmatrix} e^{E}_{x_1^E}(n) & \cdots & e^{E}_{x^E_{N_j^E}}(n)\end{bmatrix}$$

- **Cluster Tracking Errors**

$$e^{P}_{x_{\mathcal{N}^P}}(n)=-\left[\mathscr{L}^{P}(n)+\mathfrak{D}^{EP}_{in}(n)\right]x_{\mathcal{N}^P}(n)+\mathcal{A}^{EP}(n)\,x^{E}_{\mathcal{N}^E}(n)$$

$$e^{E}_{x_{\mathcal{N}^E}}(n)=-\left[\mathscr{L}^{E}(n)-\mathfrak{D}^{EP}_{out}(n)\right]x_{\mathcal{N}^E}(n)-\left(\mathcal{A}^{EP}(n)\right)^{T}x^{P}_{\mathcal{N}^P}(n)$$

where

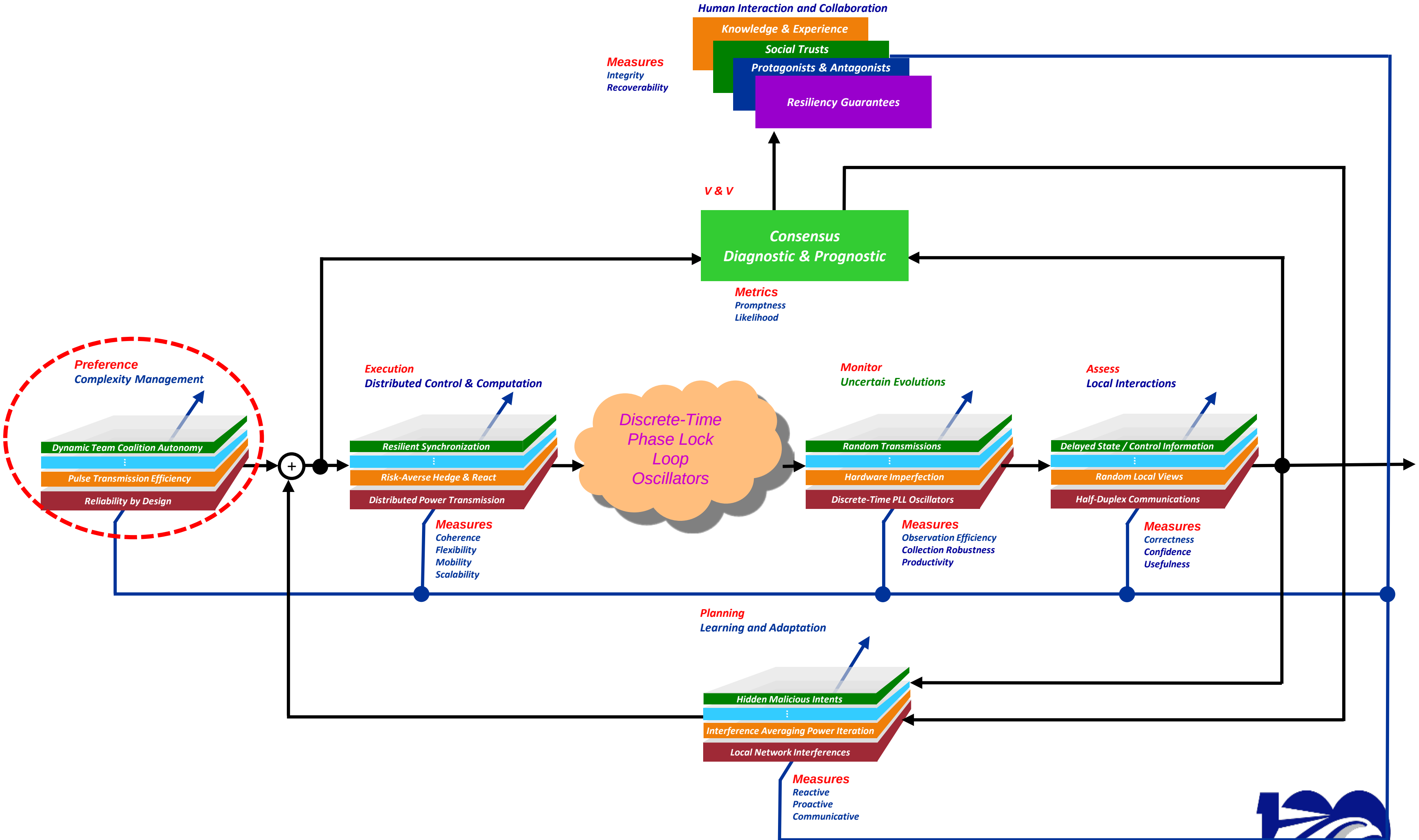
$$\mathscr{L}^{X}(n)=\mathfrak{C}^{X}(n)-\mathcal{A}^{X}(n); \qquad X=\{P,E\}$$

$$\mathfrak{D}^{EP}_{in}(n):\ \text{In-Degree Matrix}$$

$$\mathfrak{D}^{EP}_{out}(n):\ \text{Out-Degree Matrix}$$

Coupling Team Dynamics / Adversarial Interactions

Resilient Time Synchronization



Preference
Objectives and Attributes

- **Respective Tradeoffs and Payoff Couplings**

$$J_{\mathcal{N}^P,\mathcal{N}^E}(n_0) = \sum_{n=n_0+1}^{N} L\left(n, x^P_{\mathcal{N}^P}(n), x^E_{\mathcal{N}^E}(n), u^P_{\mathcal{N}^P}(n-1), u^E_{\mathcal{N}^E}(n-1)\right); \qquad N = |I|; \qquad I \subset \mathbb{N}$$

where

$$\begin{aligned} L&\left(n, x^P_{\mathcal{N}^P}(n), x^E_{\mathcal{N}^E}(n), u^P_{\mathcal{N}^P}(n-1), u^E_{\mathcal{N}^E}(n-1)\right) \\ &= \left(x^P_{\mathcal{N}^P}(n)\right)^T R_P^{-1}(n) x^P_{\mathcal{N}^P}(n) - \left(x^E_{\mathcal{N}^E}(n)\right)^T R_E^{-1}(n) x^E_{\mathcal{N}^E}(n) \\ &+ \left(u^P_{\mathcal{N}^P}(n-1)\right)^T R_P(n-1) u^P_{\mathcal{N}^P}(n-1) - \left(u^E_{\mathcal{N}^E}(n-1)\right)^T R_E(n-1) u^E_{\mathcal{N}^E}(n-1) \end{aligned}$$

Social Payouts Being Shared…

Preference
Objectives and Attributes

Let

$$\mathcal{U}^X(n) = \left\{ u^X_{\mathcal{N}^X}(n_0), u^X_{\mathcal{N}^X}(n_0+1), \ldots, u^X_{\mathcal{N}^X}(n) \right\}; \qquad X = \{P, E\}$$

$$\mathcal{V}^X(n) = \left\{ v^X_{\mathcal{N}^X}(n_0), v^X_{\mathcal{N}^X}(n_0+1), \ldots, v^X_{\mathcal{N}^X}(n) \right\}; \qquad n \in I$$

Then the unique solutions

$$x^X_{\mathcal{N}^X}(n) = x^X_{\mathcal{N}^X}\left(n; n_0, x^X_{\mathcal{N}^X}(n_0); \mathcal{U}^X(n-1), \mathcal{V}^X(n-1)\right)$$

are satisfying the cluster dynamics

$$x^X_{\mathcal{N}^X}(n+1) = A^X_{\mathcal{N}^X}(n)\, x^X_{\mathcal{N}^X}(n) + v^X_{\mathcal{N}^X}(n); \qquad X = \{P, E\}$$

$$p^X_{\mathcal{N}^X}(n+1) = A^{X,p}_{\mathcal{N}^X}(n)\, p^X_{\mathcal{N}^X}(n) + B^{X,p}_{\mathcal{N}^X}(n)\, u^X_{\mathcal{N}^X}(n)$$

Coordination and Coalition Interactions

Preference
Objectives and Attributes

$$z^T(n) = \left[\left(x^P_{\mathcal{N}^P}(n) \right)^T, \left(x^E_{\mathcal{N}^E}(n) \right)^T, \left(p^P_{\mathcal{N}^P}(n) \right)^T, \left(p^E_{\mathcal{N}^E}(n) \right)^T \right]$$

$$u_P(n) = u^P_{\mathcal{N}^P}(n); \qquad u_E(n) = u^E_{\mathcal{N}^E}(n); \qquad w^T(n) = \left[\left(v^P_{\mathcal{N}^P}(n) \right)^T, \left(v^E_{\mathcal{N}^E}(n) \right)^T \right]$$

- **Multi-Player Pursuit-Evasion Interactions**

$$z(n+1) = A(n)z(n) + B_P(n)u_P(n) + B_E(n)u_E(n) + G(n)w(n); \qquad z(n_0)$$

where

$$A(n) = \begin{bmatrix} A^P_{\mathcal{N}^P}(n) & 0 & 0 & 0 \\ 0 & A^E_{\mathcal{N}^E}(n) & 0 & 0 \\ 0 & 0 & A^{P,p}_{\mathcal{N}^P}(n) & 0 \\ 0 & 0 & 0 & A^{E,p}_{\mathcal{N}^E}(n) \end{bmatrix}; \; B_P(n) = \begin{bmatrix} 0 \\ 0 \\ B^{P,p}_{\mathcal{N}^P}(n) \\ 0 \end{bmatrix}; \; B_E(n) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ B^{E,p}_{\mathcal{N}^E}(n) \end{bmatrix}$$

$$G^T(n) = \begin{bmatrix} 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix}$$

Preference
Objectives and Attributes

$$J_{\mathcal{N}^P,\mathcal{N}^E}(n_0) = \sum_{n=n_0+1}^{N} L\left(n, z(n), u_P(n), u_E(n)\right)$$

$$= \sum_{n=n_0+1}^{N} \left[z^T(n) Q(n-1) z(n) + u_P^T(n-1) R_P(n-1) u_P(n-1) - u_E^T(n-1) R_E(n-1) u_E(n-1) \right]$$

where

$$Q(n) = \begin{bmatrix} T(n) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \qquad T(n) = \begin{bmatrix} T_{11}(n) & T_{12}(n) \\ T_{12}^T(n) & T_{22}(n) \end{bmatrix}$$

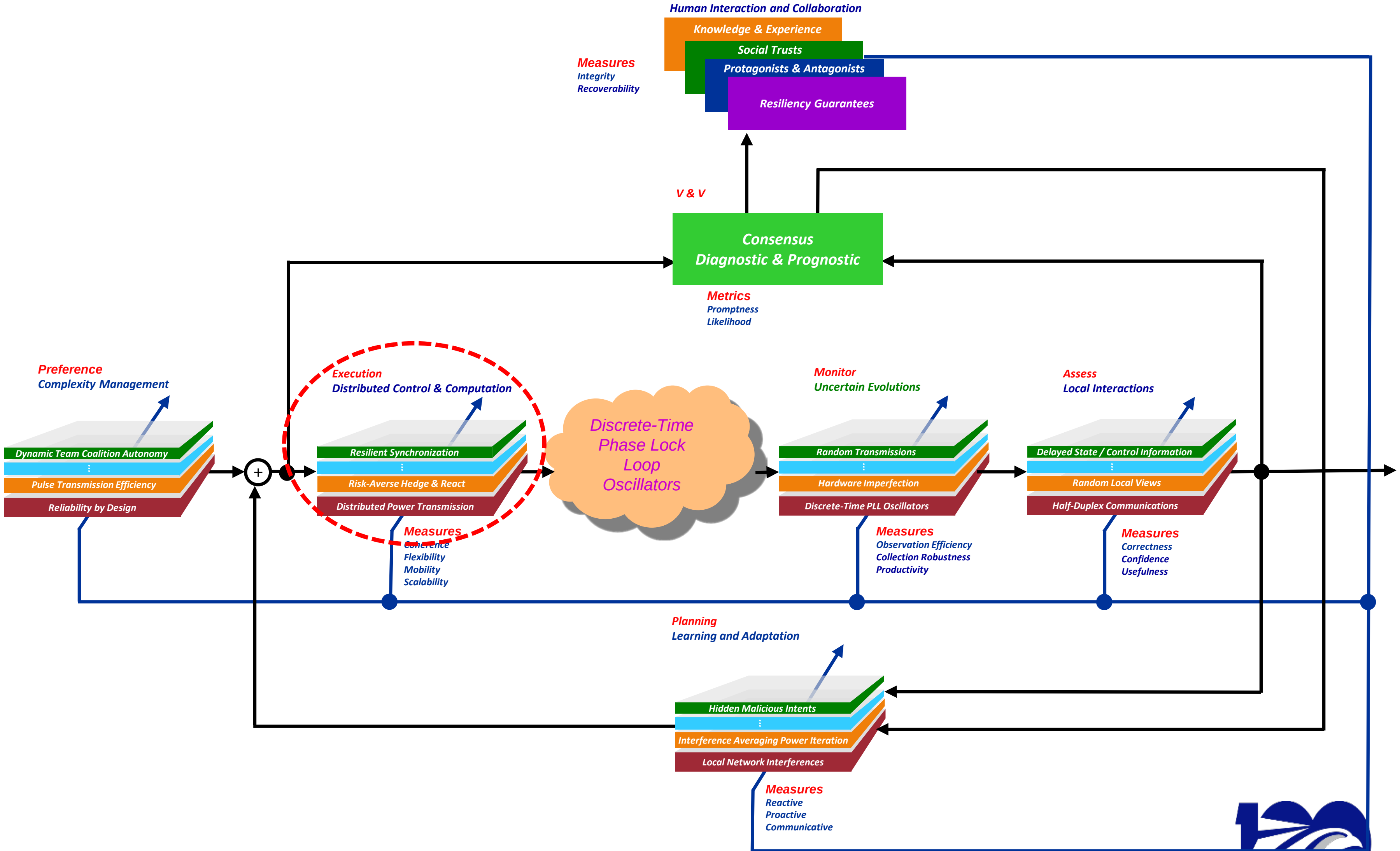
$$T_{11}(n) = \left[\mathscr{L}^P(n) + \mathfrak{D}_{in}^{EP}(n)\right]^T R_P^{-1}(n) \left[\mathscr{L}^P(n) + \mathfrak{D}_{in}^{EP}(n)\right] + \mathscr{A}^{EP}(n) R_E^{-1}(n) \left(\mathscr{A}^{EP}(n)\right)^T$$

$$T_{12}(n) = -\left[\mathscr{L}^P(n) + \mathfrak{D}_{in}^{EP}(n)\right]^T R_P^{-1}(n) \mathscr{A}^{EP}(n) + \mathscr{A}^{EP}(n) R_E^{-1}(n) \left[\mathscr{L}^E(n) - \mathfrak{D}_{out}^{EP}(n)\right]$$

$$T_{22}(n) = \left(\mathscr{A}^{EP}(n)\right)^T R_P^{-1}(n) \mathscr{A}^{EP}(n) + \left[\mathscr{L}^E(n) - \mathfrak{D}_{out}^{EP}(n)\right]^T R_E^{-1}(n) \left[\mathscr{L}^E(n) - \mathfrak{D}_{out}^{EP}(n)\right]$$

Balance of Intra-Team and Inter-Team Payouts

Resilient Time Synchronization



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Execution
Risk Hedges and React

Due to the process noise sequences

$$\mathcal{V}^{X}(n)=\left\{v_{\mathcal{N}^X}^{X}(n_0),v_{\mathcal{N}^X}^{X}(n_0+1),\ldots,v_{\mathcal{N}^X}^{X}(n)\right\}; \qquad n\in I; \qquad X=\{P,E\}$$

$$\mathcal{W}(n)=\left\{\mathcal{V}^{P}(n),\mathcal{V}^{E}(n)\right\}$$

Then it is necessary to consider the expected payoff

$$J\left(\left\{u_P(n)\right\}_{n=n_0}^{N-1},\left\{u_E(n)\right\}_{n=n_0}^{N-1};z(n_0)\right)$$

$$=E_w\left\{\sum_{n=n_0}^{N-1}\left[z^T(n+1)Q(n)z(n+1)+u_P^T(n)R_P(n)u_P(n)-u_E^T(n)R_E(n)u_E(n)\right]\right\}$$

Social Payouts Subjected to Probability Distributions

Execution
Risk Hedges and React

- **Dynamic Programming Recursion**

$$V_n\left(z(n)\right)=\min_{u_P(n)}\max_{u_E(n)}E_{w(n)}\left\{\begin{matrix}z^T(n+1)Q(n)z(n+1)+u_P^T(n)R_P(n)u_P(n)-u_E^T(n)R_E(n)u_E(n)\\+V_{n+1}\left(A(n)z(n)+B_P(n)u_P(n)+B_E(n)u_E(n)\right)\end{matrix}\right\}$$

$$V_N\left(z(N)\right)=0$$

- **Value Function of the Form**

$$V_n\left(z(n)\right)=z^T(n)P(n)z(n)+p(n)$$

Dealing with Pursuit-Evasion Struggles

Execution
Risk Hedges and React

- **Forward Recursive Matrix-Valued Equation**

$$
\begin{aligned}
&P(n+1) = A^T(n)\{P(n) - P(n)[B_P(n)S_B^{-1}(P(n))B_P^T(n) + B_P(n)S_B^{-1}(P(n))B_P^T(n) \\
&P(n)B_E(n)[R_E(n) - B_E^T(n)P(n)B_E(n)]^{-1}B_E^T(n) + B_E(n)[R_E(n) - B_E^T(n)P(n)B_E(n)]^{-1} \\
&B_E^T(n)P(n)B_P(n)S_B^{-1}(P(n))B_P^T(n) + B_E(n)[R_E(n) - B_E^T(n)P(n)B_E(n)]^{-1}B_E^T(n)P(n) \\
&B_P(n)S_B^{-1}(P(n))B_P^T(n)P(n)B_E(n)[R_E(n) - B_E^T(n)P(n)B_E(n)]^{-1}B_E^T(n) + B_E(n)[B_E^T(n) \\
&P(n)B_E(n) - R_E(n)]^{-1}B_E^T(n)]P(n)\}A(n) + Q(n); \qquad P(n_0) = Q(n_0)
\end{aligned}
$$

and

$$
\begin{aligned}
S_B^{-1}(P(n)) = {} & B_P^T(n)P(n)B_P(n) + R_P(n) \\
& + B_P^T(n)P(n)B_E(n)\left[R_E(n) - B_E^T(n)P(n)B_E(n)\right]^{-1}B_E^T(n)P(n)B_P(n)
\end{aligned}
$$

- **Forward Recursive Scalar-Valued Equation**

$$
p(n+1) = p(n) + Tr\left\{G^T(n)P(n)G(n)Q_W\right\}; \qquad p(n_0) = 0
$$

Execution
Pursuit-Evasion Decision Policy

- **Self-Enforcing and Robust Equilibrium**

$$u_P^*(n) = -S_P^{-1}\left(P(N-n-1)\right)B_P^T(n)\{I + P(N-n-1)B_E(n)$$
$$[R_E(n) - B_E^T(n)P(N-n-1)B_E(n)]^{-1}B_E(n)\}P(N-n-1)A(n)z(n)$$

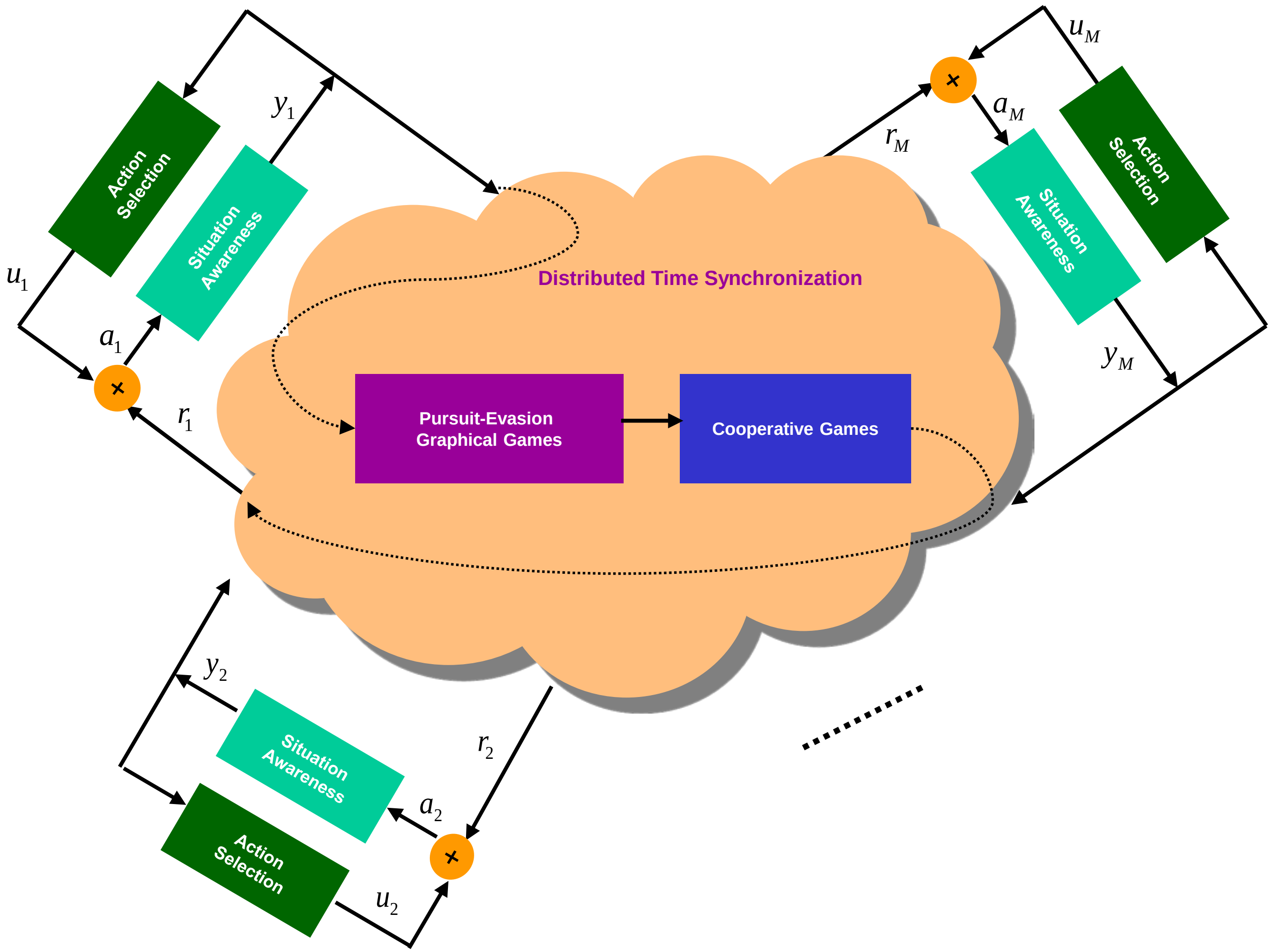
$$u_E^*(n) = \left[R_E(n) - B_E^T(n)P(N-n-1)B_E(n)\right]^{-1}B_E^T(n)\{I - P(N-n-1)B_P(n)$$
$$S_B^{-1}\left(P(N-n-1)\right)B_P^T(n)[I + P(N-n-1)B_E(n)[R_E(n) - B_E^T(n)P(N-n-1)$$
$$B_E(n)]^{-1}B_E^T(n)]\}P(N-n-1)A(n)z(n)$$

- **The Value of Pursuit-Evasion Game**

$$V_0\left(z(n_0)\right) = z^T(n_0)\left[P(N) - Q(N)\right]z(n_0)$$

No need to wait for the counterpart's change happening first - No Delay

Execution
Resilient Time Synchronization



Summary

- Graph & game-theoretical framework for physical-layer time synchronization based on
  - Coupled discrete-time oscillators
  - Timing disagreement and misinformation
  - Random transmit selection for peer sampling
- Pulse-coupled synchronization enabled by
  - Diversity reception and power weightings
  - Resilient time synchronization against random sample path realizations and active denials
- Future work involving
  - Social trusts among interacting clocks

Questions?