IEEE CCAA Workshop 2019

Assured SATCOM: Control-Theoretic System Concepts for Joint Transmission

Integrity ★ Service ★ Excellence

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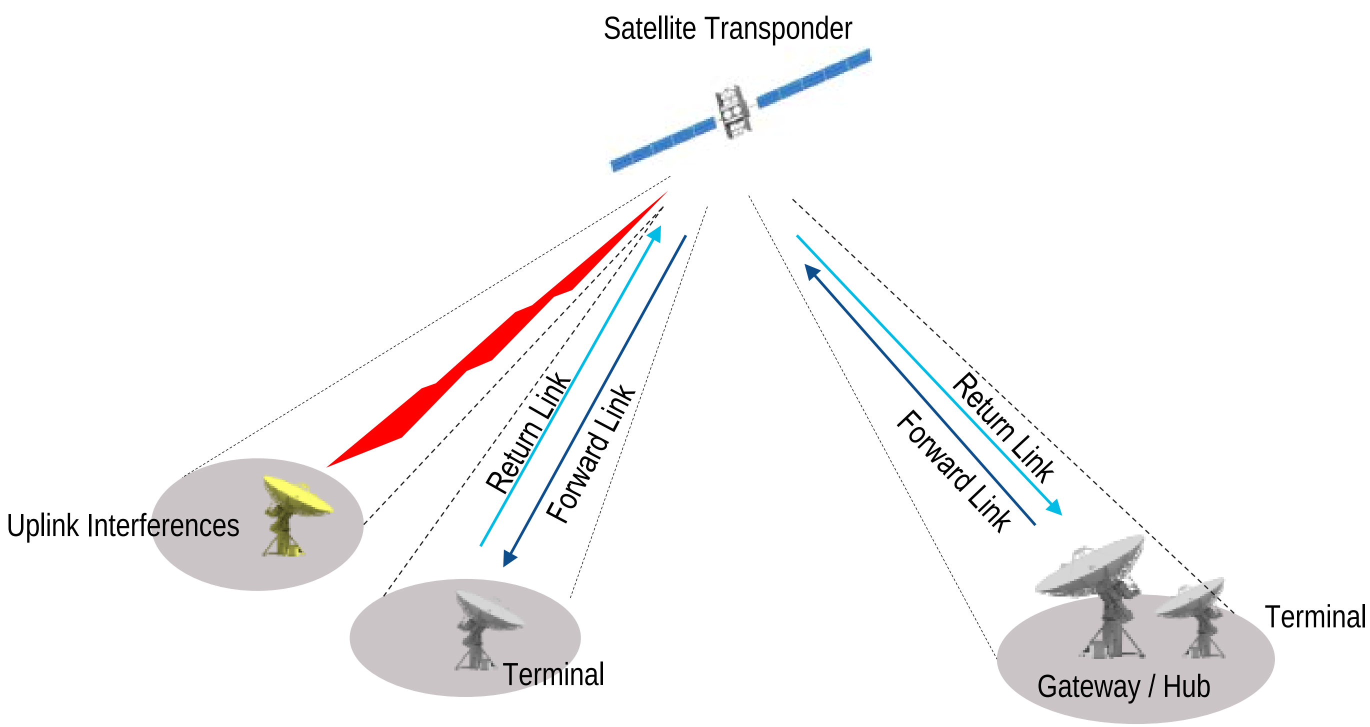
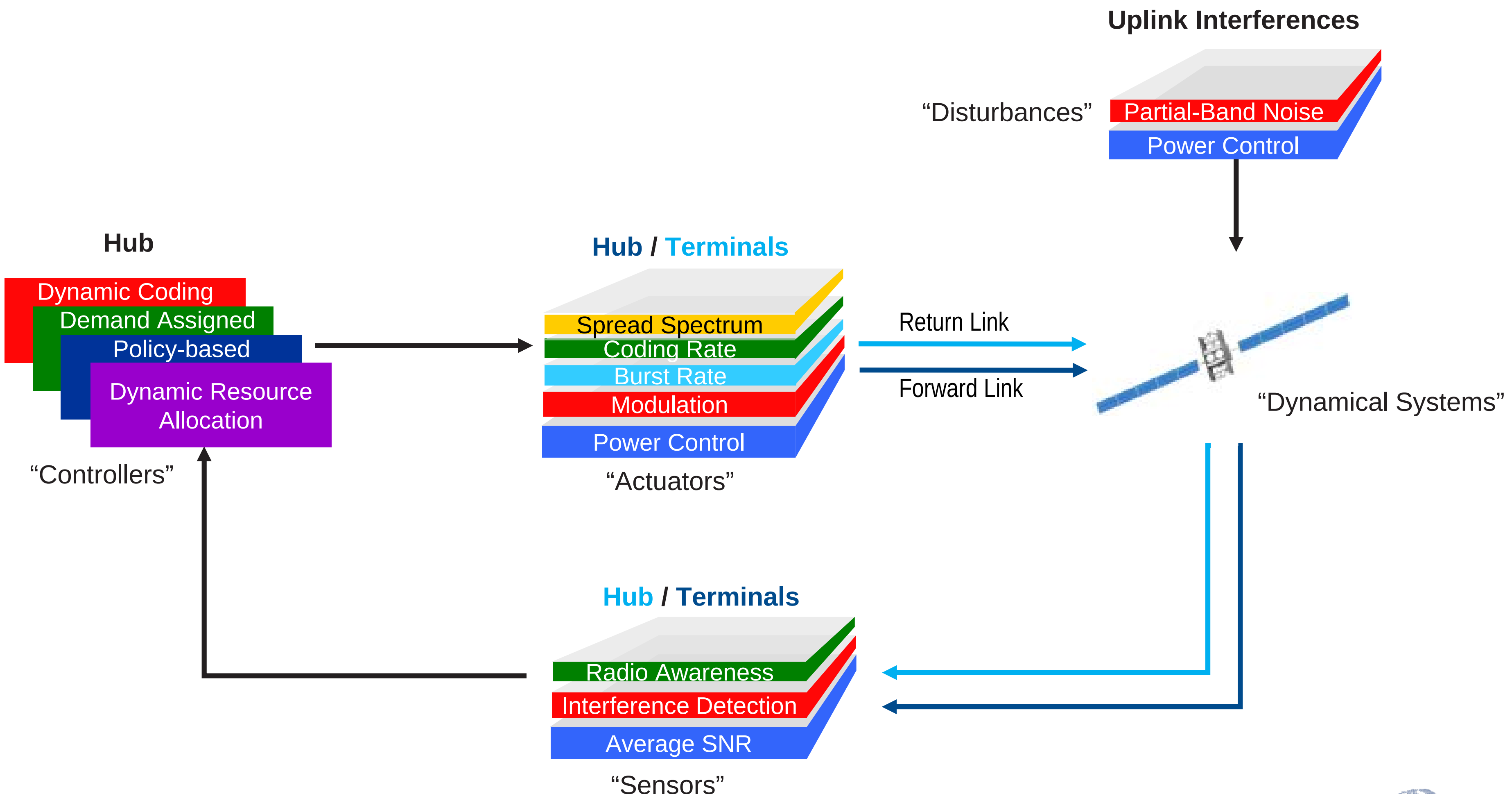
SATCOM: Satellite Communications

Traditional SATCOM Architecture



- Hub collects terminal reports from deployed terminals
- Hub provides dynamic resource management

Standard Hub / Terminal Operations



Vision

Excellence in Control Theory Research and Education to Enhance Productivity for SATCOM

Research Goals

- Embodying SATCOM values (resource sharing, cost per bit, etc.)
- Reducing complexity of payloads and user equipment

Education Goals

- Prepare responsible innovators across disciplines leveraging control-theoretical thinking and technology

Vision

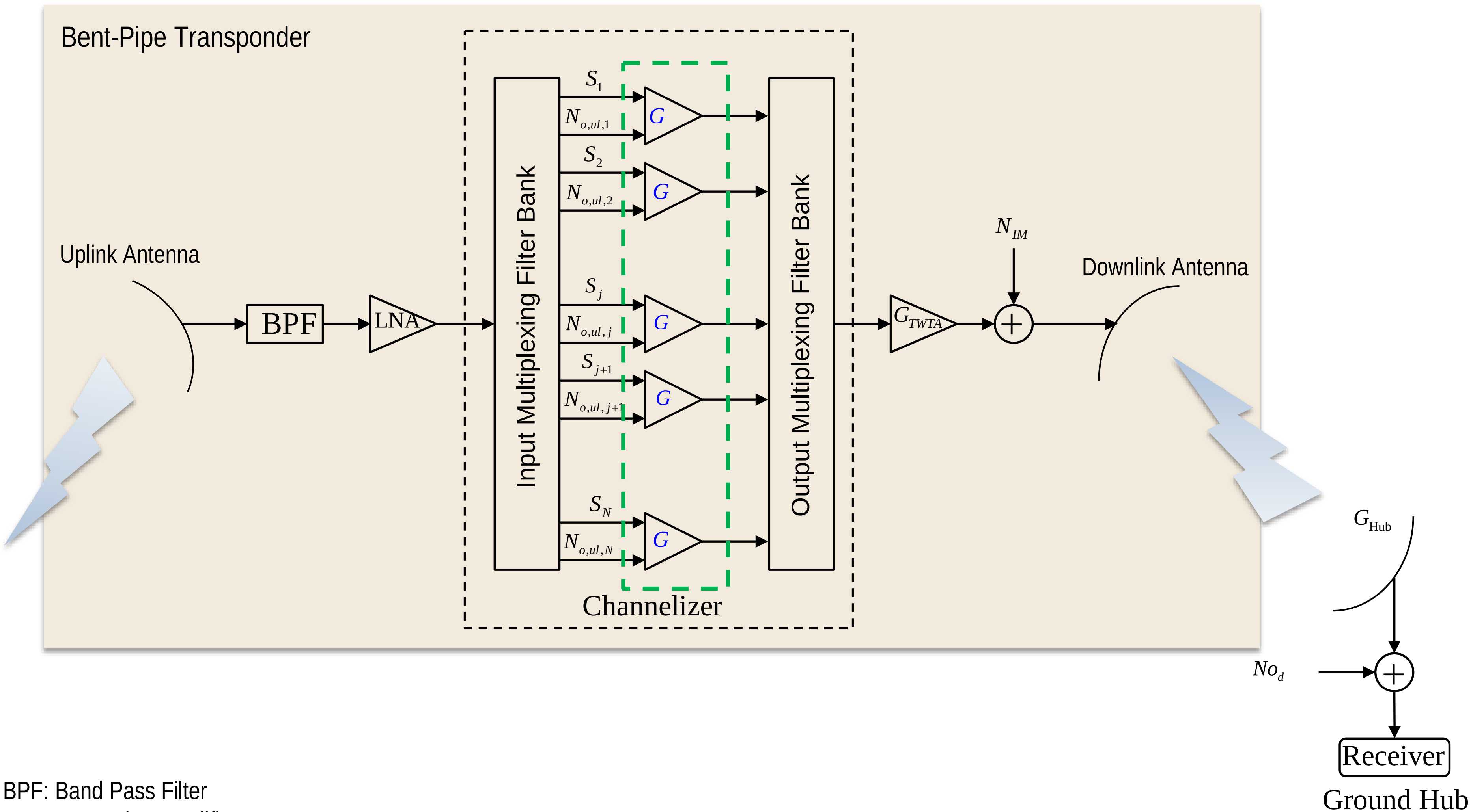
Principles for Assured SATCOM

- Good systems must be well-behaved and robust
- Good systems must be able to explain themselves
- Good systems must start to embed key values
- Good system must augment operators, enabling operators to do more

Expected Contributions

- Design criteria addressing operational needs
  - Control-theoretic approach to joint transmission of multi-carriers subject shared satellite transponders
- Technical contributions are agnostic to
  - Transponder power robbing effects
  - Wideband carriers and random losses
- Operational focus
  - Draw benefits of power sharing among multi-carriers
  - Address operational gaps of future satellite transponders
  - Manage tradeoffs between link supportability

Shared Satellite Transponders



BPF: Band Pass Filter
LNA: Low Noise Amplifier
TWTA: Traveling Wave Tube Amplifier

Transponded Link Information

- Total Signal Power at Hub

$$P_{h,s} = \sum_{i=1}^{N} S_{h,i}$$

- Maximum Total Downlink C/No

$$C2No_d^{\max} = \frac{P_{h,s}}{No_d}$$

- Maximum Uplink C/No

$$C2No_{u,i}^{\max} = C2No_{u,i} \frac{P_{t,i}^{\max}}{P_{t,i}}$$

$S_{h,i}$: Signal carrier power levels from SNR estimation

No_d : Noise power spectrum density at hub receiver

Explainability

Pre-Qualification for Supportability

- Minimum Downlink C/No at Hub Receiver

$$C2No_{d,i}^{\min} = \left[C2No_{req,i}^{-1} - \left[\left[C2No_{u,i}^{\max} \right]_{dB} - LM_{u,i} \right]_R^{-1} \right]^{-1}, \qquad i = 1, \ldots, N$$

- Minimum Total Downlink C/No and Qualification

$$\Delta_d^{\min} = \sum_{i=1}^{N} \left[\left[C2No_{d,i}^{\min} \right]_{dB} + LM_{d,i} \right]_R ; \qquad Q_d = 1 - \frac{C2No_d^{\max}}{\Delta_d^{\min}}$$

- Transponded Link and Qualification

$$\delta_i = \left[\left[C2No_{u,i}^{\max} \right]_{dB} - LM_{u,i} \right]_R^{-1} + \left[\left[C2No_d^{\max} \right]_{dB} - LM_{d,i} \right]_R^{-1} ; \qquad q_i = 1 - \frac{C2No_{req,i}^{-1}}{\delta_i}$$

- Multi-Objective Pre-Qualification

$$m_{u,i} = \varepsilon Q_d + (1 - \varepsilon) q_i ; \qquad 0 < \varepsilon < 1$$

Explainability

Uplink C/No Dynamics

$$\left[C2No_{u,i}\right]_{dB}(n+1)=\left[C2No_{u,i}\right]_{dB}(n)+\mu_i\left\{u_{C2No,i}(n)-m_{u,i}(n)\left[C2No_{u,i}\right]_{dB}(n)\right\};\quad i=1,\ldots,N$$

- Uplink Margin Distributions

$$LM_{u,i}(n+1)=LM_{u,i}(n)+w_{u,i}(n);\quad i=1,\ldots,N$$

$$\Pr\left(w_{u,i}(n)\le LM_{u,i}(n)\right)=0.99$$

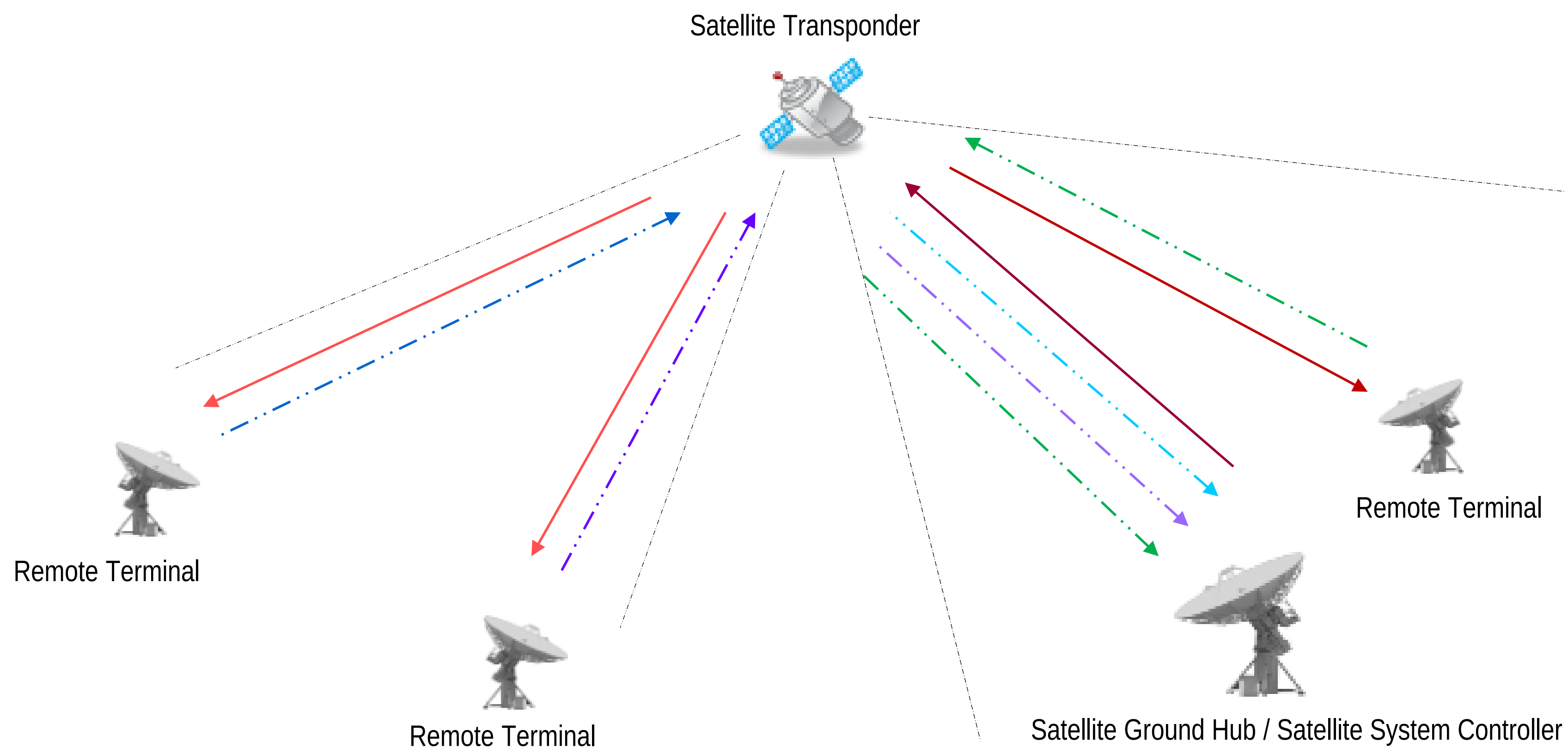
- State-space Model for Transponded Links

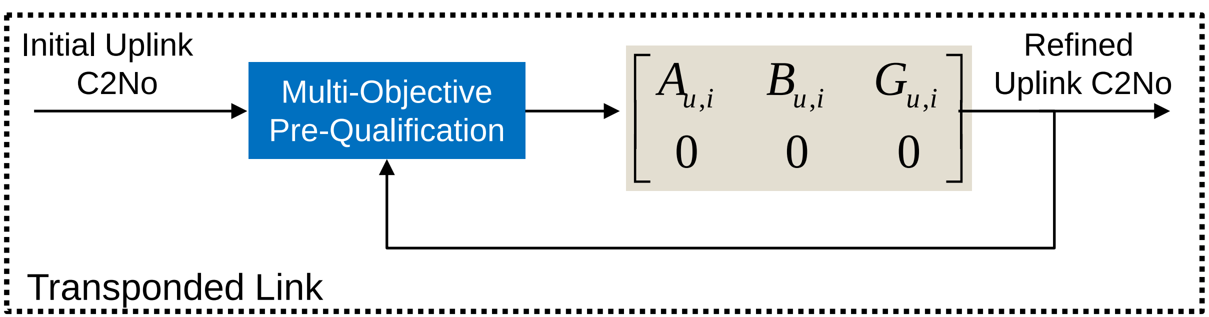
$$\underbrace{\begin{bmatrix}\left[C2No_{u,i}\right]_{dB}(n+1)\\ LM_{u,i}(n+1)\\ 1\end{bmatrix}}_{x_{u,i}(n+1)}=\underbrace{\begin{bmatrix}1-\mu_i m_{u,i}(n) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}}_{A_{u,i}}\underbrace{\begin{bmatrix}\left[C2No_{u,i}\right]_{dB}(n)\\ LM_{u,i}(n)\\ 1\end{bmatrix}}_{x_{u,i}(n)}+\underbrace{\begin{bmatrix}1\\0\\0\end{bmatrix}}_{B_{u,i}}\underbrace{u_{C2No,i}(n)}_{u_i(n)}+\underbrace{\begin{bmatrix}0\\1\\0\end{bmatrix}}_{G_{u,i}}w_{u,i}(n)$$

$$x_{u,i}(n+1)=A_{u,i}x_{u,i}(n)+B_{u,i}u_i(n)+G_{u,i}w_{u,i}(n);\quad i=1,\ldots,N$$

Explainability

Uplink C/No Dynamics





Key Values

Bent-Pipe Transponders

- Transponder Amplifier Bandwidth

$$BW_{TA}\ (Hz)$$

- Uplink Noise Power Robbing Effects

$$\left[BW_{TA}\right]_{dB} = 10\log_{10} BW_{TA}$$

- Mapping Ratio between Uplink and Downlink C2No

$$K\left[\left[C2No_{d,i^*}^{\min}\right]_{dB} + LM_{d,i^*}\right]_R = \left[\left[C2No_{u,i^*}^{\max}\right]_{dB} - LM_{u,i^*}\right]_R$$

$$K\left[\left[C2No_{d,i}^{\min}\right]_{dB} + LM_{d,i}\right]_R \le \left[\left[C2No_{u,i}^{\max}\right]_{dB} - LM_{u,i}\right]_R$$

Key Values

Qualification for Supportability

- Final Total Downlink C2No for Qualification

$$\sum_{i=1}^{N} K^{-1}\left[\left[C2No_{u,i}\right]_{dB} + LM_{u,i}\right]_R + K^{-1}BW_{TA} \overset{?}{<} C2No_d^{\max}$$

Or

$$\sum_{i=1}^{N} \underbrace{\left[-K^{-1} \quad K^{-1} \quad -\frac{K^{-1}}{N}\left[BW_{TA}\right]_{dB} + \frac{1}{N}\left[C2No_d^{\max}\right]_{dB}\right]}_{H_{u,i}} \underbrace{\begin{bmatrix} \left[C2No_{u,i}\right]_{dB} \\ LM_{u,i} \\ 1 \end{bmatrix}}_{x_{u,i}} \overset{?}{>} 0$$

- Supportability Qualification Measure

$$x_N^T Q_N x_N = \left(\sum_{i=1}^{N} H_{u,i} x_{u,i}\right)^2 = \underbrace{\begin{bmatrix} x_{u,1}^T & x_{u,2}^T & \cdots & x_{u,N}^T \end{bmatrix}}_{x_N^T} \underbrace{\begin{bmatrix} H_{u,1}^T \\ H_{u,2}^T \\ \vdots \\ H_{u,N}^T \end{bmatrix} \begin{bmatrix} H_{u,1} & H_{u,2} & \cdots & H_{u,N} \end{bmatrix}}_{Q_N} \underbrace{\begin{bmatrix} x_{u,1} \\ x_{u,2} \\ \vdots \\ x_{u,N} \end{bmatrix}}_{x_N}$$

Key Values

Satellite Ground Hub

- State-Space Model for Aggregate Transponded Links

$$\underbrace{\begin{bmatrix} x_{u,1}(n+1) \\ x_{u,2}(n+1) \\ \vdots \\ x_{u,N}(n+1) \end{bmatrix}}_{x_N(n+1)} = \underbrace{\begin{bmatrix} A_{u,1}(n) & 0 & \cdots & 0 \\ 0 & A_{u,2}(n) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & A_{u,N}(n) \end{bmatrix}}_{A_N} \underbrace{\begin{bmatrix} x_{u,1}(n) \\ x_{u,2}(n) \\ \vdots \\ x_{u,N}(n) \end{bmatrix}}_{x_N(n)}$$

$$+ \underbrace{\begin{bmatrix} B_{u,1} & 0 & \cdots & 0 \\ 0 & B_{u,2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & B_{u,N} \end{bmatrix}}_{B_N} \underbrace{\begin{bmatrix} u_1(n) \\ u_2(n) \\ \vdots \\ u_N(n) \end{bmatrix}}_{u_N(n)} + \underbrace{\begin{bmatrix} G_{u,1} & 0 & \cdots & 0 \\ 0 & G_{u,2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & G_{u,N} \end{bmatrix}}_{G_N} \underbrace{\begin{bmatrix} w_{u,1}(n) \\ w_{u,2}(n) \\ \vdots \\ w_{u,N}(n) \end{bmatrix}}_{w_N(n)}$$

Augmentation

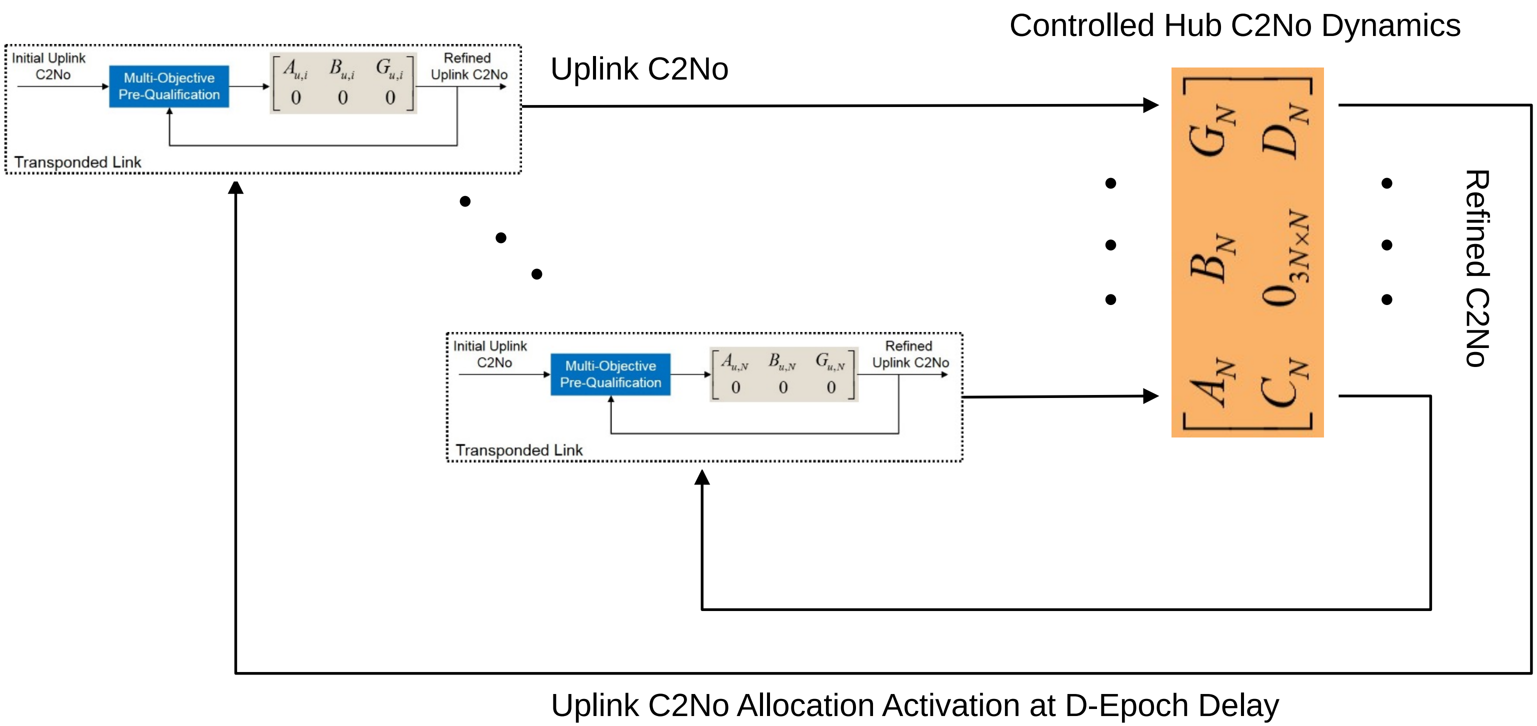
+

- Noisy Observations

$$y_N(n) = C_N x_N(n) + D_N v_N(n)$$

Satellite Ground Hub

- Feedback Structure of Uplink C2No Allocation



Key Values

Satellite Ground Hub

- Successive tracking of actual total downlink C2No, inherent transponder power robbing effects and maximum total downlink C2No allowed
- Successive adjustments of uplink C2No sequences

$$J_N(n_0) = \sum_{k=n_0+1}^{L} \left\{ x_N^T(k) Q_N x_N(k) + u_N^T(k) R_N u_N(k) \right\}$$

subject to

$$x_N(n+1) = A_N x_N(n) + B_N u_N(n) + G_N w_N(n); \qquad x_N(n_0)$$
$$y_N(n) = C_N x_N(n) + D_N v_N(n)$$

Key Values

Satellite Ground Hub

- State Estimates based on Kalman Filtering

$$\hat{x}_N(n|n) = A_N \hat{x}_N(n-1|n-1) + B_N u_N(n-1) + L_N(n)\left\{y_N(n) - C_N \hat{x}_N(n-1|n-1)\right\}$$

$$L_N(n) = P_N(n|n-1) C_N^T \left[C_N P_N(n|n-1) C_N^T + D_N V_N D_N^T\right]^{-1}$$

$$P_N(n|n-1) = A_N P_N(n-1|n-1) A_N^T + G_N W_N G_N^T$$

$$P_N(n-1|n-1) = L_N(n-1) D_N V_N D_N^T L_N^T(n-1) + \left[I - L_N(n-1) C_N\right] P_N(n-1|n-2) \left[I - L_N(n-1) C_N\right]^T$$

$$\hat{x}_N(n_0|n_0) = x_N(n_0) + P_N(n_0) C_N^T \left[C_N P_N(n_0) C_N^T + D_N V_N D_N^T\right]^{-1} \left[y_N(n_0) - C_N x_N(n_0)\right]$$

$$E\left\{w_N(n)\right\} = 0; \qquad E\left\{w_N(n) w_N^T(n+\tau)\right\} = W_N \delta(\tau)$$

$$E\left\{v_N(n)\right\} = 0; \qquad E\left\{v_N(n) v_N^T(n+\tau)\right\} = V_N \delta(\tau)$$

Robustness

Minimal-Cost-Variance Control

$$x_N(n+1) = A_N x_N(n) + B_N u_N(n) + G_N w_N(n); \qquad x_N(n_0)$$

$$J_N(n_0) = \sum_{k=n_0+1}^{L} \left\{ x_N^T(k) Q_N x_N(k) + u_N^T(k) R_N u_N(k) \right\}$$

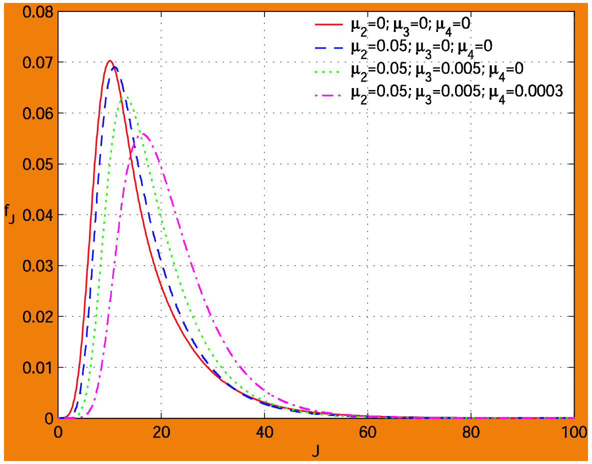
- Measures of Performance Risk and Uncertainties

$$\text{Mean: } \kappa_1 = E\{J_N\}$$

$$\text{Variance: } \kappa_2 = E\{J_N^2\} - E^2\{J_N\}$$

$$\text{Skewness: } \kappa_3 = E\{J_N^3\} - 3E\{J_N^2\}E\{J_N\} - 2E^3\{J_N\}$$

$$\text{Flatness: } \kappa_4 = E\{J_N^4\} - 4E\{J_N^3\}E\{J_N\} - 3E^2\{J_N^2\} + 12E\{J_N^2\}E^2\{J_N\} - 6E^4\{J_N\}$$



Variance, Skewness, Kurtosis... as Higher-Order Stochastic Dominance

Robustness

Minimal-Cost-Variance Control

- Minimizing Cost Variance

$$E\left\{J_N^2\left(n_0\right)|Z_N\left(n_0\right)\right\}-E^2\left\{J_N\left(n_0\right)|Z_N\left(n_0\right)\right\}$$

while its mean subject to the constraint

$$E\left\{J_N\left(n_0\right)|Z_N\left(n_0\right)\right\}=h_N\left(n_0,Z_N\left(n_0\right)\right);\qquad Z_N\left(n_0\right)=\left\{x_N\left(n_0\right)\right\}$$

- Linear-Quadratic Control Problem Class

$$h_N\left(n_0,Z_N\left(n_0\right)\right)=m_N\left(n_0\right)+x_N^T\left(n_0\right)M_N\left(n_0\right)x_N\left(n_0\right)$$

where

$$h_N\left(n_0,Z_N\left(n_0\right)\right)>\sigma_N\left(n_0,Z_N\left(n_0\right)\right)\equiv\inf_{u_N\left(n_0\right),\ldots,u_N\left(L-1\right)}E\left\{J_N\left(n_0\right)|Z_N\left(n_0\right)\right\}$$

Robustness

Minimal-Cost-Variance Control

- Correct-by-Design Adjustments for Uplink C2No

$$u_N^*(n) = K_N^*(n)\hat{x}_N(n|n)$$

$$K_N^*(n) = -\left[B_N^T \Lambda_N^*(n) B_N + \lambda_N(n) R_N\right]^{-1} B_N^T \Lambda_N^*(n) A_N$$

subject to the mean constraint

$$\Lambda_N^*(n) = S_N^*(n) G_N W_N G_N^T S_N^*(n) + \frac{1}{4} V_N^*(n+1) + \lambda_N(n) S_N^*(n)$$

$$S_N^*(n) = Q_N(n) + M_N^*(n+1); \qquad n_0 \le n \le L-1$$

$$M_N^*(n) = \left(K_N^*(n)\right)^T R_N K_N^*(n) + \left(A_N^*(n)\right)^T S_N^*(n) A_N^*(n); \qquad M_N^*(L) = 0$$

$$m_N^*(n) = m_N^*(n+1) + Tr\left\{S_N^*(n) G_N W_N G_N^T\right\}; \qquad m_N^*(L) = 0$$

Robustness

Optimal Cost Variances

$$VC_N^*\left(n+1,Z_N\left(n+1\right)\right)=v_N^*\left(n+1\right)+x_N^T\left(n\right)V_N^*\left(n+1\right)x_N\left(n\right)$$

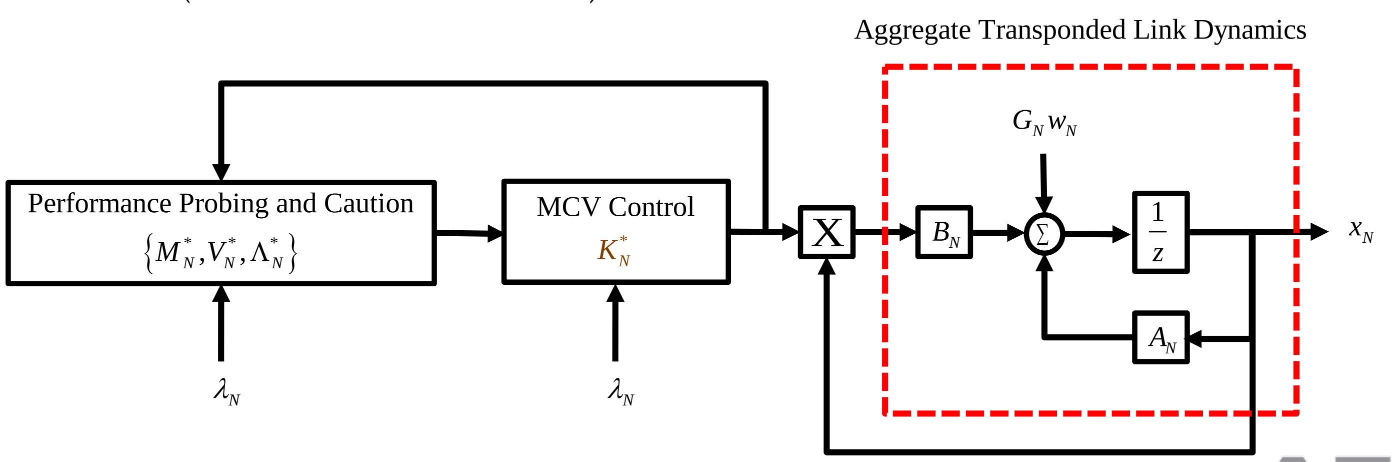
provided

$$A_N^*\left(n\right)=A_N+B_N K_N^*\left(n\right)$$

$$V_N^*\left(n\right)=\left(A_N^*\left(n\right)\right)^T\left[4S_N^*\left(n\right)G_N W_N G_N^T S_N^*\left(n\right)+V_N^*\left(n+1\right)\right]A_N^*\left(n\right);\qquad V_N^*\left(L\right)=0$$

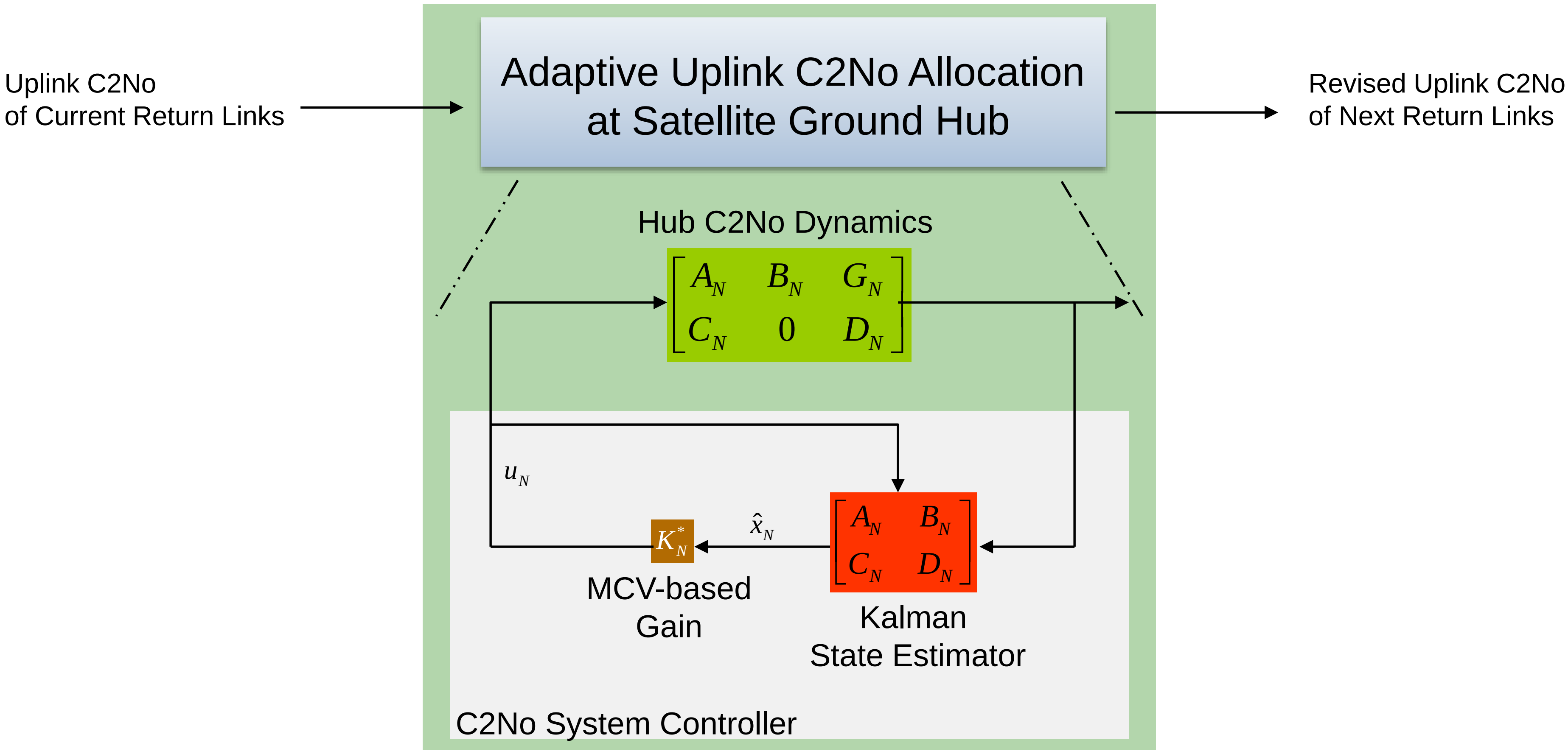
$$v_N^*\left(n\right)=v_N^*\left(n+1\right)+Tr\left\{V_N^*\left(k+1\right)G_N W_N G_N^T\right\}+E\left\{\left(w_N^T\left(n\right)G_N^T S_N^*\left(n\right)G_N w_N\left(n\right)\right)^2\right\}$$

$$-\left(Tr\left\{S_N^*\left(n\right)G_N W_N G_N^T\right\}\right)^2;\qquad v_N^*\left(L\right)=0$$



Robustness

Satellite Ground Hub



Summary

- System and method of optimizing shared satellite transponders and terminals by means of
  - Dynamic C2No resource allocation
  - Compensation for transponder power robbing and distributions of uplink and downlink margins
- Risk-averse uplink C2No control enabled by
  - Performance uncertainty forecast and management
  - Resilient C2No control sequences hedging against transponder power robbing effects and uplink / downlink uncertainties while subject to maximum total downlink C2No
- Future work involving
  - User-case simulation analysis